# HOMEWORK 9

NAME:____________________

Problem 1: Integrate the function $H(x,y,z) = yz$ over the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the cone $z = \sqrt{x^2 + y^2}$.

*Date*: April 1, 2025.

Problem 2: Compute the flux of the vector field

$$\mathbf{F} = y^2\,\mathbf{i} + xz\,\mathbf{j} - \mathbf{k}$$

outward (normal away from the $z$-axis) through the conical surface $S$ defined by

$$z = \sqrt{2x^2 + y^2}, \quad 0 \le z \le 2.$$

Problem 3: Let $S$ be the portion of the cylinder $y = e^x$ in the first octant that projects parallel to the $x$-axis onto the rectangle $R_{yz}$:

$$1 \le y \le 2, \quad 0 \le z \le 1$$

in the $yz$-plane. Let $\mathbf{n}$ be the unit vector normal to $S$ that points away from the $yz$-plane. Draw a picture of $S$ and find the flux of the field

$$\mathbf{F}(x,y,z) = -2\mathbf{i} + 2y\mathbf{j} + z\mathbf{k}$$

across $S$ in the direction of $\mathbf{n}$.

Problem 4: Recall that for a parametrized surface $\mathbf{r} : \Omega \to S$ with coordinates $(u,v)$ on $\Omega$, the area is computed by

$$\text{Area}(S) = \iint_\Omega \|\mathbf{r}_u \times \mathbf{r}_v\| du dv.$$

Check that Area$(S)$ is independent of the choice of parametrization: Suppose that $\mathbf{g} : D \to S$ is another parametrization of $S$ with coordinate $(s,t)$ on $\Omega'$, then

$$\iint_\Omega \|\mathbf{r}_u \times \mathbf{r}_v\| du dv = \iint_{\Omega'} \|\mathbf{g}_s \times \mathbf{g}_t\| ds dt.$$

(Hint: By our standing assumptions on parametrized surfaces, $\mathbf{g}, \mathbf{r}$ are bijective, and $\mathbf{g}^{-1} \circ \mathbf{r}$ and $\mathbf{r}^{-1} \circ \mathbf{g}$ are continuously differentiable. Use change of variable formula for integrals.)

Problem 5: Verify Stokes' Theorem for the vector field

$$\mathbf{F} = 2xy\mathbf{i} + x\mathbf{j} + (y+z)\mathbf{k}$$

and the surface $S$ defined by $z = 4 - x^2 - y^2$, $z \geq 0$, oriented with the unit normal $\mathbf{n}$ pointing upward. (Compute both sides of Stokes' theorem directly and check that they are equal.)