Suggested Solution to HOMEWORK 3

Problem 1

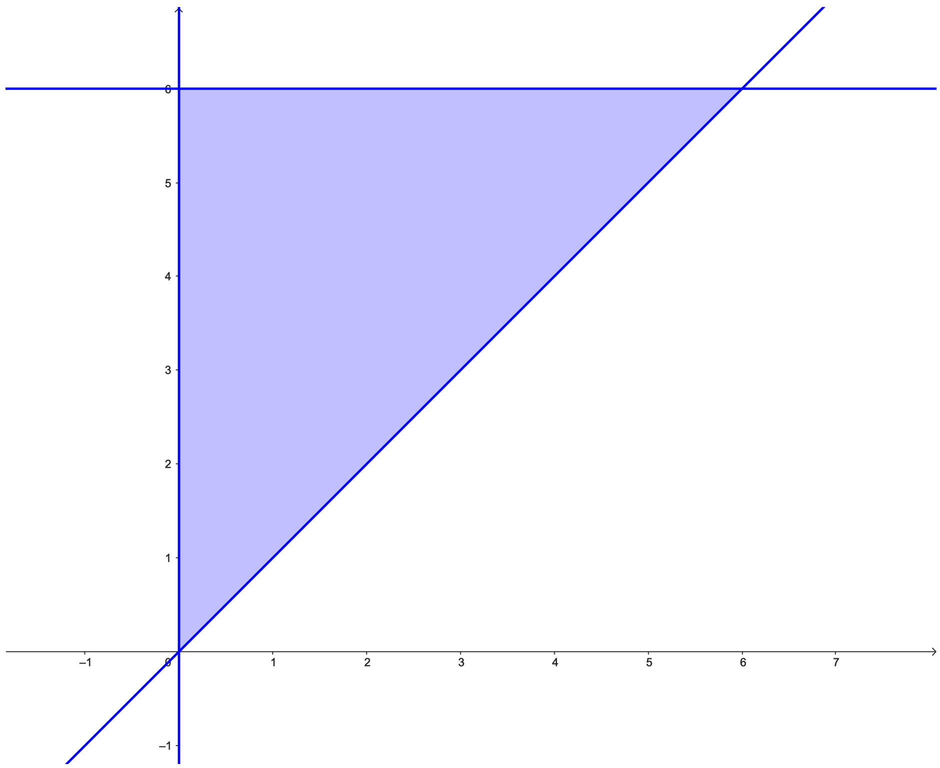
Solution.

$$
\begin{aligned}
&\int_0^{\frac{\pi}{2}} \int_0^{\sqrt{2-\sin(2u)}} r dr du \\
=&\int_0^{\frac{\pi}{2}} \left[\frac{r^2}{2}\right]_0^{\sqrt{2-\sin(2u)}} du \\
=&\int_0^{\frac{\pi}{2}} \frac{2-\sin(2u)}{2} du \\
=&\left[\frac{1}{2}\left(2u+\frac{\cos(2u)}{2}\right)\right]_0^{\frac{\pi}{2}} \\
=&\frac{1}{2}\left(\pi-\frac{1}{2}-\frac{1}{2}\right) \\
=&\frac{\pi-1}{2}
\end{aligned}
$$

□

Problem 2

Solution.



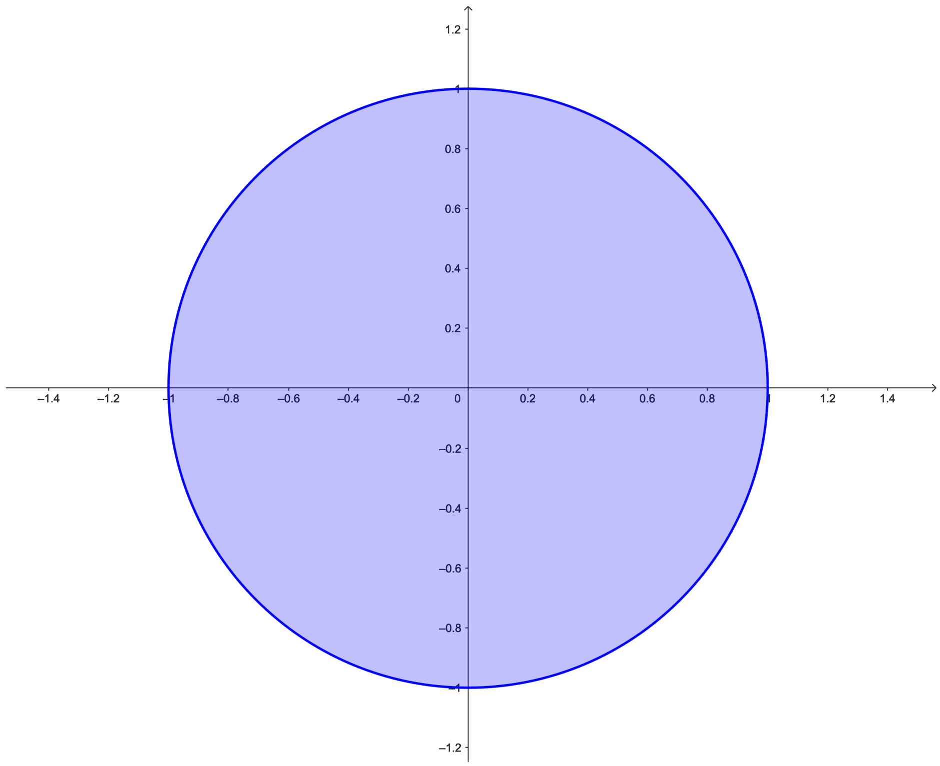
When $y = 6$, $r\sin\theta = 6$, $r = \dfrac{6}{\sin\theta}$.

$$
\begin{aligned}
&\int_0^6 \int_0^y x dx dy \\
=&\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{\frac{6}{\sin\theta}} (r\cos\theta) r dr d\theta \\
=&\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{\frac{6}{\sin\theta}} (r^2\cos\theta) dr d\theta \\
=&\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[\frac{r^3\cos\theta}{3}\right]_0^{\frac{6}{\sin\theta}} d\theta \\
=&\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{72\cos\theta}{\sin^3\theta} d\theta \\
=&\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{72}{\sin^3\theta} d\sin\theta \\
=&\left[\frac{72}{-2\sin^2\theta}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} \\
=&\left[\frac{-36}{\sin^2\theta}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} \\
=& -36 - (-36)(2) \\
=&36
\end{aligned}
$$

□

Problem 3

Solution.



$$
\begin{aligned}
&\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln\left(x^2+y^2+1\right)dxdy \\
=&\int_{0}^{2\pi}\int_{0}^{1} \ln\left(r^2+1\right) r dr d\theta \\
=&\int_{0}^{2\pi}\int_{0}^{1} \frac{1}{2}\ln\left(r^2+1\right) d(r^2+1) d\theta \\
=&\int_{0}^{2\pi}\left(\left[\frac{1}{2}(\ln\left(r^2+1\right))(r^2+1)\right]_0^1 - \frac{1}{2}\int_0^1 (r^2+1) d(\ln\left(r^2+1\right))\right) d\theta \\
=&\int_{0}^{2\pi}\left(\ln 2 - \int_0^1 r dr\right) d\theta \\
=&\int_{0}^{2\pi}\left(\ln 2 - \left[\frac{r^2}{2}\right]_0^1\right) d\theta \\
=&\int_{0}^{2\pi}\left(\ln 2 - \frac{1}{2}\right) d\theta \\
=&2\pi\left(\ln 2 - \frac{1}{2}\right) \\
=&\pi(2\ln 2 - 1)
\end{aligned}
$$

□

Problem 4

Solution.

$$
\begin{aligned}
&\int_0^{2\pi}\int_1^{\sqrt{e}} \frac{\ln\left(r^2\right)}{r} r dr d\theta \\
=&\int_0^{2\pi}\int_1^{\sqrt{e}} 2\ln r dr d\theta \\
=&\int_0^{2\pi} 2\left(\left[r\ln r\right]_1^{\sqrt{e}} - \int_1^{\sqrt{e}} r d(\ln r)\right) d\theta \\
=&\int_0^{2\pi} 2\left(\frac{\sqrt{e}}{2} - \int_1^{\sqrt{e}} 1 dr\right) d\theta \\
=&\int_0^{2\pi} 2\left(\frac{\sqrt{e}}{2} - \sqrt{e} + 1\right) d\theta \\
=&\int_0^{2\pi} \left(2 - \sqrt{e}\right) d\theta \\
=&2\pi\left(2 - \sqrt{e}\right)
\end{aligned}
$$

□

Problem 5

Solution.

$$
\begin{aligned}
&\int_0^{2\pi}\int_0^{\frac{4\theta}{3}} r dr d\theta \\
=&\int_0^{2\pi}\left[\frac{r^2}{2}\right]_0^{\frac{4\theta}{3}} d\theta \\
=&\int_0^{2\pi}\frac{16\theta^2}{2(9)} d\theta \\
=&\frac{8}{9}\left[\frac{\theta^3}{3}\right]_0^{2\pi} \\
=&\frac{8}{9}\frac{8\pi^3}{3} \\
=&\frac{64\pi^3}{27}
\end{aligned}
$$

□