

## AUG-09 GROUP 4 EXERCISES

### DETERMINANT

**Exercise 0.1.** We want to deduce the formula for the determinant

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc.$$

It turns out that

$$\left| \det \begin{bmatrix} a & b \\ c & d \end{bmatrix} \right| = \text{Area of the quadrilateral PQRS}$$

where

$$\begin{aligned} P &= (0, 0), \\ Q &= (a, c), \\ R &= (b, d), \\ S &= (a + b, c + d). \end{aligned}$$

Show the following for all real  $a, b, c, d$ .

- (1) Argue geometrically that

$$\left| \det \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix} \right| = |ad|.$$

- (2) Argue geometrically that

$$\left| \det \begin{bmatrix} 0 & b \\ c & 0 \end{bmatrix} \right| = |bc|.$$

- (3) Argue geometrically that

$$\left| \det \begin{bmatrix} a & 0 \\ c & d \end{bmatrix} \right| = \left| \det \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix} \right|.$$

[Hint: Draw two quadrilaterals, what do you notice?]

- (4) Argue geometrically that

$$\left| \det \begin{bmatrix} 0 & b \\ c & d \end{bmatrix} \right| = \left| \det \begin{bmatrix} 0 & b \\ c & 0 \end{bmatrix} \right|.$$

- (5) Argue geometrically that

$$\left| \det \begin{bmatrix} a + b & b \\ c + d & d \end{bmatrix} \right| = \left| \det \begin{bmatrix} a & b \\ c & d \end{bmatrix} \right|.$$

[Hint: Draw two quadrilaterals]

- (6) For any  $\lambda \in \mathbb{R}$ , argue geometrically that

$$\left| \det \begin{bmatrix} a + \lambda b & b \\ c + \lambda d & d \end{bmatrix} \right| = \left| \det \begin{bmatrix} a & b \\ c & d \end{bmatrix} \right|.$$

[Hint: Adapt the argument from (5)]

(7) Show that

$$\left| \det \begin{bmatrix} a & b \\ c & d \end{bmatrix} \right| = |ad - bc|.$$

[Hint: Choose a suitable  $\lambda$ ]

(8) Argue why

$$\left| \det \begin{bmatrix} a & b \\ c & d \end{bmatrix} \right| = \left| \det \begin{bmatrix} b & a \\ d & c \end{bmatrix} \right|$$

without using the formula.

### MATRICES

**Exercise 0.2.** Calculate

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

**Exercise 0.3.** Calculate

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

**Exercise 0.4.** Calculate

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

**Exercise 0.5.** Calculate

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

**Exercise 0.6.** Calculate

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

**Exercise 0.7.** Calculate

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Exercise 0.8.** Calculate

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$$

**Exercise 0.9.** Calculate

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}$$

**Exercise 0.10.** Calculate

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

**Excercise 0.11.** Calculate

$$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

**Excercise 0.12.** Calculate

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

**Excercise 0.13.** Calculate

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

**Excercise 0.14.** Calculate

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

**Excercise 0.15.** Calculate

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

**Excercise 0.16.** Calculate

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

**Excercise 0.17.** Calculate

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

**Excercise 0.18.** Calculate

$$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

**Excercise 0.19.** Calculate

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

**Excercise 0.20.** Calculate

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

**Excercise 0.21.** Calculate

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

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