SAYT1134 Towards Differential Geometry
Group 3 Tutorial 3
Exercise

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If you have any questions about the lecture notes or syllabus of Test 1 after the tutorial, feel free to ask in the WhatsApp group or DM us. (The three admins in the group are the Teaching Assistants)

1 Curves

1. Determine if the function $\mathbf{r}(t) = (e^t, e^{-t})$, $t \in \mathbb{R}$, is a regular parametrized curve.

2. Determine if the function $\mathbf{r}(s) = (\cos(s), \sin(s))$, $s \in \mathbb{R}$, is a regular parametrized curve. (It is a parametrization of the unit circle)

3. Determine if the function $\mathbf{r}(t) = (\cos(t^2), \sin(t^2))$, $t \in \mathbb{R}$, is a regular parametrized curve. (Note that it is also a parametrization of the unit circle)

4. Find a parametrized curve $\mathbf{r}(t)$ whose trace is the quartic equation $y = x^4$.

5. Find a parametrized curve $\mathbf{r}(t)$ whose trace is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ such that $\mathbf{r}(t)$ runs clockwise around the circle with $\mathbf{r}(0) = (a, 0)$.

2 Arc length

1. Consider a parametrized curve $\mathbf{r}(t) = (2\ln t, 2t, \frac{1}{2}t^2)$, $t \in [1, 2]$. Find the arc length of $\mathbf{r}$.

2. Show that the curve $\mathbf{r}(s) = (\cos(\frac{s}{\sqrt{2}}), \sin(\frac{s}{\sqrt{2}}), \frac{s}{\sqrt{2}})$, $s \in \mathbb{R}$, is an arc length parametrized curve.

3. Consider $\mathbf{r}(t) = (\cos(5t), \sin(5t), 12t)$, $t \in \mathbb{R}$. What is the speed of $\mathbf{r}$? Find the arc-length parametrization $\mathbf{r}(s)$.

4. (Tricky) The equation $x^{2/3} + y^{2/3} = 1$, which is called an astroid. Parametrize the curve by $\mathbf{r}(t) = (\cos^3 t, \sin^3 t)$, $t \in [0, 2\pi)$. Find the arc length of $\mathbf{r}$.

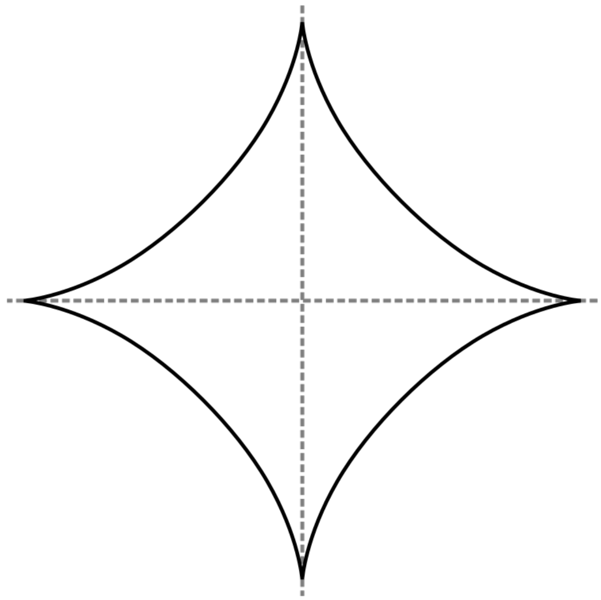

Figure 1: Asteroid

3 Curve curvature

1. Let $\mathbf{r}(t) = (t, \ln\cos t)$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$. Find the unit tangent vector, unit normal vector and curvature of the curve.