Image Processing Experiments

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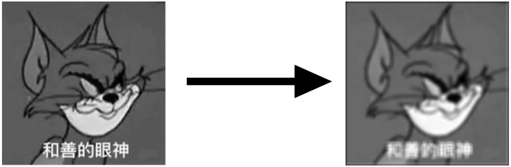
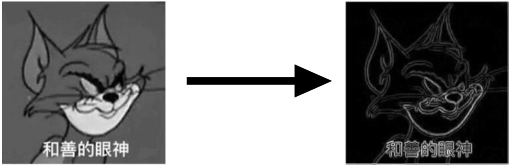
Overview

1. Image processing on the spatial domain

   1.1 Generating blurry images



   1.2 Application of Sobel operator



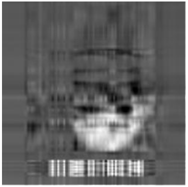
Overview (cont.)

2. Image processing on the spectral domain

   2.1 Application of SVD

(a) Original image

(b) k=2



(c) k=5

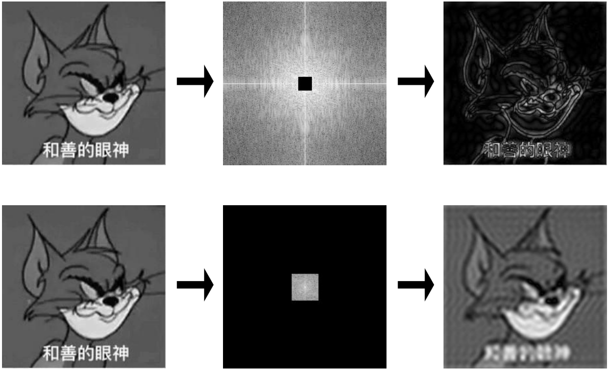
(d) k=10

(e) k=20

(f) k=50

Overview (cont.)

2.2 Application of FFT



Softwares

Scientific computation tools:

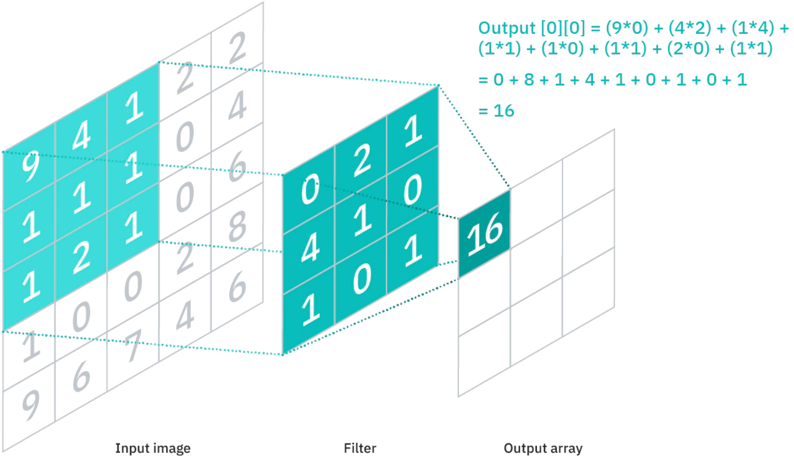
- MATLAB: MATLAB is a powerful scientific computing software developed by MathWorks. The MATLAB environment integrates graphics illustrations with precise numerical calculations, and is an easy-to-use tool for performing all kinds of computations and data visualization. CUHK Campus-wide License
- Octave: GNU Octave is an open source software primarily intended for numerical computations. Its programming language is mostly compatible with MATLAB. Octave Downloading page
- Python: Python is a general-purpose programming language. It be can utilized in scientific computing with third party libraries such as NumPy, SciPy, PyTorch, scikit-learn, TensorFlow, OpenCV, Matplotlib, etc.
- ...

The idea is to apply an average kernel to the image.

$$kernel = \frac{1}{25} \times \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \tag{1}$$

Experiment 1.1

Convolution:



Experiment 1.2: Sobel Operator

Sobel operator consists of two kernels:

$$ker1 = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix} \quad and \quad ker2 = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{pmatrix} \tag{2}$$

Suppose the input image is denoted as I, then the output image G can be computed as follows:

$$G = \sqrt{(ker1 * I)^2 + (ker2 * I)^2} \tag{3}$$

Later in this course, we would like to introduce image processing techniques in the frequency domain. Here we briefly show two examples:

- Singular Value Decomposition (SVD)
- Discrete Fourier Transform (DFT)

For any $g \in M_{m\times n}$, the **singular value decomposition (SVD)** of g is a matrix factorization given by

$$g = U\Sigma V^T$$

with $U \in M_{m\times m}$ and $V \in M_{n\times n}$ both orthogonal, and $\Sigma \in \mathbb{R}^{m\times n}$ is a diagonal matrix ($\Sigma_{ij} = 0$ if $i \neq j$) with diagonal elements $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r > 0$ with $r \leq \min(m,n)$. The diagonal elements are called the **singular values** of g.

$$g = \begin{bmatrix} | & & | \\ \vec{u}_1 & \cdots & \vec{u}_m \\ | & & | \end{bmatrix} \begin{pmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_r \end{pmatrix} \begin{bmatrix} - & \vec{v}_1^T & - \\ - & \vec{v}_2^T & - \\ & \vdots & \\ - & \vec{v}_n^T & - \end{bmatrix}.$$

For an $N \times N$ image g, the DFT of g is given by:

$$\hat{g}(m,n) = \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} g(k,l) e^{-2\pi j \frac{km+ln}{N}}.$$

Let $U_{x\alpha} = \dfrac{1}{N} e^{-2\pi j \frac{x\alpha}{N}}$ where $0 \leq x, \alpha \leq N-1$, and $U = (U_{x\alpha})_{0 \leq x, \alpha \leq N-1} \in M_{N\times N}(\mathbb{C})$. Then, U is symmetric and

$$\hat{g} = UgU.$$

In the following experiment, we will use Fast Fourier Transform (FFT).
FFT is an optimized algorithm for DFT.
The steps are as follows.

1. Use Fast Fourier Transform to obtain the spectrum of the image.
2. Perform some manipulation on the spectrum.
3. Use inverse Fast Fourier Transform to obtain the resulting image.

Here we simply use fft2, fftshift, ifft2 and ifftshift to conduct the experiment. For details, please click the hyperlinks of the 4 functions.