

**THE CHINESE UNIVERSITY OF HONG KONG**  
**Department of Mathematics**  
**MMAT 5120 Topics in Geometry 2023-24**  
**Lecture 8 practice problems**  
**1st November 2023**

- The practice problems are meant as exercise to the students. You are **NOT** required to submit your solutions, but you are encouraged to work through all of them in order to understand the course materials. The problems will be uploaded on Fridays and solutions will be uploaded on Wednesdays before the next lecture.
- Please send an email to **zdmu@math.cuhk.edu.hk** if you have any questions.

1. Show that for any two points  $p, q \in \mathbb{D}$ , there are two horocycles passing through them.
2. Recall the picture in lecture 8 page 3 about angle of parallelism. Prove the following relation between  $\theta$  and the distance  $x = |\overline{pr}|$  between  $p, r$

$$\frac{2x}{x^2 + 1} = \cos \theta$$

Remark: Here we are using Euclidean distance, which is NOT invariant under hyperbolic geometry. Later on we will introduce hyperbolic distance, and obtain another relation of this form between hyperbolic distance and angle of parallelism.