

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MMAT 5120 Topics in Geometry 2023-24
Lecture 11 practice problems
17th November 2023

- The practice problems are meant as exercise to the students. You are **NOT** required to submit your solutions, but you are encouraged to work through all of them in order to understand the course materials. The problems will be uploaded on Fridays and solutions will be uploaded on Wednesdays before the next lecture.
- Please send an email to **zdmu@math.cuhk.edu.hk** if you have any questions.

1. Show that \bar{H} is a transformation group for \mathbb{U} .
2. Every statement we have learnt about the disc model (\mathbb{D}, H) has a corresponding statement in the upper half plane model (\mathbb{U}, \bar{H}) since they are isomorphic geometries. For example, recall Q2 from practice problems set 7. Any Möbius transformation in H is either elliptic, hyperbolic or parabolic, and it is determined by the locations of the fixed points.
 - (a) Explain why any $f \in \bar{H}$ is either elliptic, hyperbolic or parabolic, using the known results from (\mathbb{D}, H) .
 - (b) Briefly describe a necessary and sufficient condition for each of the above three cases in \bar{H} , using Q2 of practice problems set 7.
 - (c) Describe another simple condition by considering fixed points, using the equation $f(z) = z$.
3. How many horocycles pass through the points $1 + i, -1 + i$ in \mathbb{U} ? What are they?
4. What is the distance between ri and si in \mathbb{U} ? (Hint: just use cross ratio.)
5. Can you deduce a relation between hyperbolic area of a hyperbolic polygon and its internal angles?