THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MMAT 5120 Topics in Geometry 2023-2024
Lecture 1 practice problems
8th September 2023

- The practice problems are meant as exercise to the students. You are **NOT** required to submit your solutions, but you are encouraged to work through all of them in order to understand the course materials. The problems will be uploaded on Fridays and solutions will be uploaded on Wednesdays before the next lecture.

- Please send an email to **zdmu@math.cuhk.edu.hk** if you have any questions.

1. Express the following complex numbers in Cartesian form $x + iy$ and polar form $re^{i\theta}$ respectively.

   (a) $(1 + i)^3$

   (b) $(1 + i)(1 - i)$

   (c) $e^{i+\pi}$

   (d) $\frac{i}{4}$

   (e) $\frac{1}{1+i}$

2. Recall from the lecture de Moivre's formula:

   $$(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta).$$

   Use this to prove the double angle formula $\cos(2\theta) = \cos^2\theta - \sin^2\theta$ and $\sin(2\theta) = 2\sin\theta\cos\theta$.

3. Let $z = re^{i\theta}$, for $r \neq 0$, what is z^{-1} in polar form? C

4. Try to find the square root of $2i$, that is try to solve for z so that $z^2 = 2i$, using each of the following methods:

   (a) Write $z = x + iy$ in Cartesian form, compare real and imaginary part of both sides of the equation $(x + iy)^2 = 2i$.

   (b) Write $z = re^{i\theta}$ in polar form, and solve for $(re^{i\theta})^2 = 2i$ by expressing $2i$ in polar form as well.

5. Does de Moivre's formula make sense for non-integer exponent? For example, what happens if we take $n = \frac{1}{2}$, is $(\cos\theta + i\sin\theta)^{\frac{1}{2}} = \cos(\frac{1}{2}\theta) + i\sin(\frac{1}{2}\theta)$ true? What happens if we take $\theta = 0$ and $\theta = 2\pi$?

   (The real question I want you to think about is, what does it mean to take square root on the complex plane.)