THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MMAT 5120 (2023-24, Term 1)
Topics in Geometry
Homework 2
Due Date: 30th November 2023

We denote by $\mathbf{i}$ the imaginary unit $\sqrt{-1}$ and by $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ the open unit disk.

1. Find the area of the hyperbolic triangle in $\mathbb{D}$ with vertices at -1, 1, $\frac{\mathbf{i}}{\sqrt{3}}$.

2. Find the area of the hyperbolic triangle in $\mathbb{D}$ with vertices at 0, $\mathbf{i}$, $2 - \sqrt{3}$.

3. Show that the sum of exterior angles of a hyperbolic polygon is $2\pi + A$, where A is the hyperbolic area of the polygon.

4. Find the area of a hyperbolic equilateral triangle, each side of which has hyperbolic length $\cosh^{-1}(1 + \sqrt{2})$. (*Hint:* Use the Sine and Cosine Rules.)

5. Let $\triangle ABC$ be a hyperbolic isosceles triangle right angled at C.

   (a) Suppose the area of $\triangle ABC$ is $\frac{\pi}{6}$. Show that the length of the edge BC (or equivalently AC) is given by $\ln(\sqrt{2} + \sqrt{3})$.

   (b) Show that the distance from the vertex C to the side AB is less than $\ln(1 + \sqrt{2})$.

   (*Hint:* Use the Sine and Cosine Rules.)