MMAT 5010 Linear Analysis (2023-24): Homework 6
Deadline: 16 Mar 2024

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.

1. Let $X := \mathbb{R}^2$ be a two dimensional real vector space and let A be the matrix $\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$. Define a mapping $T : X \to X$ by $Tx = Ax$ for $x \in X$. Suppose that X is endowed with the $\|\cdot\|_1$-norm, that is $\|x\|_1 := |x_1| + |x_2|$ for $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in X$. Find $\|T\|$.

2. Recall that c_{00} denotes the finite sequence space which is equipped with the $\|\cdot\|_1$-norm. Let $T : c_{00} \to c_{00}$ be the linear map given by

$$T(x)(k) := kx(k)$$

   for $k = 1, 2...$ and $x \in c_{00}$. Show that T is a discontinuous map.

3. The left shift operator is given by $T : \ell^2 \to \ell^2$ by $T(x_1, x_2, x_3, ...) := (x_2, x_3, x_4, ...)$ for $(x_1, x_2, ..) \in \ell_2$. Find $\|T\|$.

$* * *$ **End** $* * *$