MMAT 5010 Linear Analysis (2023-24): Homework 7
Deadline: 23 Mar 2024

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.

1. Let $X := \mathbb{R}^N$ be the N-dimensional real vector space with the usual norm, that is, $\|x\| := \sqrt{x_1^2 + \cdots + x_N^2}$ for $x = (x_1, ..., x_N)$. For each $x, y \in \mathbb{R}^N$, put $T(x)(y) := \sum_{k=1}^{N} x(k)y(k)$. Show that T is an isometric isomorphism from $\mathbb{R}^N$ onto its dual space.

2. Let X be a normed space and let $0 \neq x_0 \in X$. Show that there is $f \in X^*$ such that $f(x_0) = 1$ and $\|f\| = 1/\|x_0\|$.

$* * *$ **End** $* * *$