MMAT 5010 Linear Analysis (2023-24): Homework 4
Deadline: 24 Feb 2024

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.

1. Let $B(X, Y)$ denote the space of all bounded linear operators between the normed spaces X and Y. Show that if we put $\|T\| := \sup\{\|Tx\| : x \in X; \|x\| \leq 1\}$, then $\|\cdot\|$ is a norm function on $B(X, Y)$.

2. Let $T : X \to Y$ and $G : Y \to Z$ be the bounded linear operators between normed spaces. Show that the composition $\|G \circ T\| \leq \|G\|\|T\|$.

3. Let $X = C[0, 1]$ be the Banach space endowed with the sup-norm. Define the operator $T : X \to \mathbb{R}$ by $Tf := \int_0^1 f(x)dx$ for $f \in X$. Find $\|T\|$.

$***$ **Happy Year of Dragon** $***$