

## MMAT 5010 Linear Analysis (2023-24): Homework 3

Deadline: 17 Feb 2024

### Important Notice:

- ♣ The answer paper must be submitted before the deadline.
- ♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.

1. Let  $\mathbb{K}^n$  be a  $n$ -dimension column vector space. Let  $A$  be a  $n \times n$  matrix. Show that the map  $x \in \mathbb{K}^n \mapsto Ax \in \mathbb{K}^n$  is continuous with respect to any norm  $\|\cdot\|$  defined on  $\mathbb{K}^n$ .
2. Let  $X$  be a normed space. For each element  $(x, y) \in X \oplus X$ , put  $\|(x, y)\|_1 := \|x\| + \|y\|$  and  $\|(x, y)\|_\infty := \max(\|x\|, \|y\|)$ . Show that  $\|\cdot\|_1$  and  $\|\cdot\|_\infty$  are equivalent norms on  $X \oplus X$ .
3. Show that if  $(x_n)$  is a convergent sequence in  $\ell_1$ , then it is also a convergent sequence with respect to the  $\|\cdot\|_\infty$ . Give an example of a sequence to show that the converse of this statement is not true.

**\*\*\* Happy Year of Dragon\*\*\***