MMAT 5010 Linear Analysis (2023-24): Homework 2
Deadline: 03 Feb 2024

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.

1. Let X be a normed space. Show that the addition $(x, y) \in X \times X \mapsto x + y \in X$ and the scalar multiplication $(\alpha, x) \in \mathbb{R} \times X \mapsto \alpha x \in X$ both are continuous maps, that is, whenever $x_n \to x$ and $y_n \to y$ in X and the scalars $\alpha_n \to \alpha$, we have $x_n + y_n \to x + y$ and $\alpha_n x_n \to \alpha x$.

2. Let X be a normed space. Show that X is a Banach space if and only if the unit sphere $S_X := \{x \in X : \|x\| = 1\}$ of X is complete, that is, every Cauchy sequence (x_n) in S there is an element $x \in S$ such that $\lim_n x_n = x$.

$* * *$ **End** $* * *$