MMAT 5010 Linear Analysis (2023-24): Homework 1
Deadline: 26 Jan 2024

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.

1. Let $(X, \|\cdot\|_X)$ and $(Y, \|\cdot\|_Y)$ be the normed spaces. Let $X \oplus Y := \{(x, y) : x \in X; y \in Y\}$ denote the direct sum of X and Y. For each element $(x, y) \in X \oplus Y$, put $\|(x, y)\|_1 := \|x\|_X + \|y\|_Y$.
   (a) Show that $\|\cdot\|_1$ is a norm function on $X \oplus Y$.
   (b) Show that if X and Y both are Banach spaces then the space $X \oplus Y$ under the norm $\|\cdot\|_1$ is also a Banach space.

2. Let $\ell^\infty[0, 1] := \{f : [0, 1] \to \mathbb{R} : f \text{ is a bounded function on } [0, 1]\}$. Let

$$\|f\|_\infty := \sup_{x\in[0,1]} |f(x)|$$

for $f \in \ell^\infty[0, 1]$. Show that $(\ell^\infty[0, 1], \|\cdot\|_\infty)$ is a Banach space.

$* * *$ **End** $* * *$