

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH4240 - Stochastic Processes - 2023/24 Term 2

A Historical Note

It is said on Pages 464–465 in the book *Introduction to Probability* by Grinstead and Snell (1997):

“Markov chains were introduced by Andrei Andreyevich Markov (1856–1922) and were named in his honor. He was a talented undergraduate who received a gold medal for his undergraduate thesis at St. Petersburg University. Besides being an active research mathematician and teacher, he was also active in politics and participated in the liberal movement in Russia at the beginning of the twentieth century. In 1913, when the government celebrated the 300th anniversary of the House of Romanov family, Markov organized a counter-celebration of the 200th anniversary of Bernoulli’s discovery of the Law of Large Numbers.

Markov was led to develop Markov chains as a natural extension of sequences of independent random variables. In his first paper, in 1906, he proved that for a Markov chain with positive transition probabilities and numerical states the average of the outcomes converges to the expected value of the limiting distribution (the fixed vector). In a later paper he proved the central limit theorem for such chains. Writing about Markov, A. P. Youschkevitch remarks:

Markov arrived at his chains starting from the internal needs of probability theory, and he never wrote about their applications to physical science. For him the only real examples of the chains were literary texts, where the two states denoted the vowels and consonants. 1 ed. C. C. Gillespie (New York: Scribner’s Sons, 1970. pp. 124–130.)

In a paper¹ written in 1913, Markov chose a sequence of 20,000 letters from Pushkin’s Eugene Onegin to see if this sequence can be approximately considered a simple chain. He obtained the Markov chain with transition matrix

$$\begin{pmatrix} .128 & .872 \\ .663 & .337 \end{pmatrix}$$

The fixed vector for this chain is (.432, .568), indicating that we should expect about 43.2 percent vowels and 56.8 percent consonants in the novel, which was borne out by the actual count.”

¹A. A. Markov, An Example of Statistical Analysis of the Text of Eugene Onegin Illustrating the Association of Trials into a Chain, *Bulletin de l’Academie Imperiale des Sciences de St. Petersburg*, Ser. 6, vol. 7 (1913), pp. 153–162.