Homework 7

All questions of this homework are optional, so **you are NOT required to hand in the solutions**. However, you are strongly encouraged to try to do all selected questions on or before April 22 Monday. The reference solution to this homework will be uploaded on April 23 Tuesday.

Exercises (Chapter 3, Page 107): 11, 12, 13, 16, 17, 18, 19, 21,

and

Q1. Suppose that the time to repair a machine is an exponentially distributed random variable with mean 2h.
  (a) What is the probability that the repair takes more than 2h?
  (b) What is the probability that the repair takes more than 5h given that it has taken more than 3h?

Q2. Alice and Betty enter a beauty parlor simultaneously, Alice to get a manicure and Betty to get a haircut. Suppose that the time for a manicure (haircut) is exponentially distributed with mean 20 (30) min.
  (a) What is the probability that Alice gets done first?
  (b) What is the expected amount of time until Alice and Betty are both done?

Q3. Ron, Sue, and Ted arrive at the beginning of a professor's office hours. The amount of time they will stay is exponentially distributed with means of 1, 1/2, and 1/3 hour.
  (a) What is the expected time until only one student remains?
  (b) For each student find the probability they are the last student left.
  (c) What is the expected time until all three students are gone?

Q4. A telephone booth has 1 telephone and 2 waiting spaces. Suppose people come in as a Poisson process with rate 2 per minute. Each one use the phone for 1 minute in average, and the usage time is an exponential random variable. Let $X(t)$ denote the number of people in the booth at time t.
  (a) Find the rate matrix for the process.
  (b) In the long term, what is the probability that there are 2 persons in the booth?

Q5. Suppose that the arrival rate at a checkout counter is 2 customers per minute. A single clerk is working at the counter and the service time is an exponential random variable with mean time 1/2 minute. However, if there are 3 customers or more, then someone will come to help and the service time reduces to a mean of 1/3 minute.
  (a) Set up the queuing model in an infinite matrix.

    (b) What is the stationary distribution of the queue?

    (c) In the long term, what is probability that there are 4 customers waiting (including the one being served)?

Q6. Suppose there are three computers in an office that are subject to failure and repair. The failure of each computer is an exponential distribution with average once in 50 days; there is only on repairman and the repair time for each computer is an exponential distribution with mean 2 days. In the long term, what is the probability that all three computers are functioning?