Assignment 9

1. Consider the convex problem

$$\min f(x) \text{ subject to } g_i(x) \leq 0, i = 1, \ldots, m$$

Assume that $x^* \in \mathbb{R}^n, \lambda^* \in \mathbb{R}^m$ satisfy the KKT conditions

$$\begin{aligned} g_i(x^*) \leq 0, i = 1, \ldots m \\ \lambda_i^* \geq 0, i = 1, \ldots, m \\ \lambda_i^* g_i(x^*) = 0, i = 1, \ldots, m \\ \nabla f(x^*) + \sum \lambda_i^* \nabla g_i(x^*) = 0 \end{aligned}$$

Show that

$$\langle \nabla f(x^*), x - x^* \rangle \geq 0$$

for all feasible x.

2. Consider

$$\begin{gathered} \max_{(x,y,z) \in \mathbb{R}^3} 2x + 3y + 2z \\ \text{subject to } x^2 + y^2 + z^2 = 1, x + y + z \geq 0 \end{gathered}$$

(a) Show that an optimal solution exists.
(b) Write down the KKT conditions.
(c) Solve the KKT conditions.

3. Let A be an $n \times n$ real symmetric matrix. Consider

$$\min_{\|x\|=1} \langle x, Ax \rangle$$

(a) Write down the KKT conditions.
(b) Assuming the KKT conditions are necessary and sufficient, show that the optimal value is an eigenvalue of A.