Assignment 7

1. Consider the following problem

$$\min x^2 + 1 \text{ subject to } (x-2)(x-4) \leq 0$$

(a) Find the feasible set, optimal value and the optimal solution.
(b) Write down the Lagrangian $L(x, \lambda)$. Find the dual function q.
(c) Solve the dual problem. Does strong duality hold?

2. Consider

$$\min_{x \in \mathbb{R}} x$$
$$\text{subject to } x^2 \leq 0$$

(a) Write down the dual problem. Hence, show that there is no duality gap.
(b) Show that there is no dual optimal solution.
(This example shows that dual optimal solution may not exist, even if there is no duality gap.)

3. Consider

$$\min x_1^2 + x_2^2$$
$$\text{subject to } (x_1 - 1)^2 + (x_2 - 1)^2 \leq 1, (x_1 - 1)^2 + (x_2 + 1)^2 \leq 1$$

(a) Find the feasible set, optimal solution x^* and optimal value p^*. (b) Write down the KKT conditions. Can you find λ_1^*, λ_2^* such that $x^*, (\lambda_1^*, \lambda_2^*)$ satisfy the KKT conditions?