# Assignment 6

**1.** Give the subdifferential sets of the following functions:

(a) $I(x) = \begin{cases} 0, & x \in X, \\ \infty, & x \notin X, \end{cases}$ where $X \subseteq \mathbb{R}^N$ is a convex set.

(b) $f(x) = \|x\|_2, x \in \mathbb{R}^N$.

**2.** Let $f : X \to \mathbb{R}$ be convex and differentiable and $X$ be a convex set. Then $0 \in \partial f(x_0)$ if and only if

$$\langle \nabla f(x_0), x - x_0 \rangle \geq 0, \forall x \in X.$$

**3.** Let $f(x) = \sum_{i=1}^{k} L(x^T u_i) + \|x\|_2^2, x \in \Omega$, where $L : \mathbb{R} \to \mathbb{R}$ is convex, $\Omega \subseteq \mathbb{R}^N$ is compact and $u_i \in \mathbb{R}^N$. Show that

(a) $\min_{x \in \Omega} f$ can be achieved at some $x^* \in \Omega$;

(b) $x^*$ is a linear combination of $\{u_1, \ldots, u_k\}$.