Assignment 5

1. Prove that

(a) For any f (convex or not), x_0 is a minimizer of f if and only if $0 \in \partial f(x_0)$;

(b) Let $f : \mathbb{R} \to \mathbb{R}$ be convex and differentiable. Then $0 \in \partial f(x_0)$ if and only if $\langle \nabla f(x_0), x - x_0 \rangle \geq 0$ for any $x \in \mathbb{R}$.

2. Let f be a convex function. Prove that x_0 is a local minimum of f, then x_0 is a global minimum of f.

3. It is known that for a convex function f and a closed and bounded set K contained in $\text{ri}(\text{dom} f)$, f is Lipschitz continuous on K. Using counter examples to explain why these three conditions (1) closedness, (2) boundedness, and (3) $K \subset \text{ri}(\text{dom} f)$ are essential.