Homework 4

1. Show the convexity of following functions:

(a) $f(x,y) = \frac{x^2}{y}, x \in \mathbb{R}, y > 0$.

(b) $f(x,y) = \ln\left(e^x + e^y\right), x, y \in \mathbb{R}$.

2. Assume $f : \Omega \subset \mathbb{R}^n \to \mathbb{R}$ is twice differentiable. Determine which of the following implications are true. Briefly give your reason if the implication is true and give a counter example if not.

(a) $\nabla^2 f(x) \succ 0, \forall x \in \Omega \Leftrightarrow f$ is strictly convex on Ω.

(b) A function f is strictly convex on $\Omega \subseteq \mathbb{R}^n$ if and only if

$$f(y) > f(x) + \nabla f^T(x)(y - x), \forall x, y \in \Omega, x \neq y$$

(c) f is strongly convex if and only if there exists $m > 0$ such that

$$f(y) \geq f(x) + \nabla^T f(x)(y - x) + m\|y - x\|^2, \forall x, y \in \mathrm{dom}(f)$$

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(d) f is strongly convex if and only if there exists $m > 0$ such that

$$\nabla^2 f(x) \succeq mI, \forall x \in \mathrm{dom}(f)$$

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3. (a) Give the subdifferential of $f(x) = \max\left\{x^2 + 2x - 3, x^2 - 2x - 3, 0\right\}$;

(b) Show that the subgradient of $f(x) = \sqrt{x}$ does not exist at $x = 0$.