Homework 3

1. Show that the following operations preserve convexity:

(a) Let $f : \mathbb{R}^N \to \mathbb{R}$ be convex, then $g(x) = f(Ax), A \in \mathbb{R}^{N\times N}$, is convex.

(b) Let $f_\lambda, \lambda \in \Lambda$ be a group of convex functions. Then $\sup_{\lambda\in\Lambda}$ is convex. (Hint: In Assignment 2, we prove the case for two functions. You can follow a similar procedure either using the definition or using the epigraph.)

2. (a) It is known that the following implication holds:

$$\text{strong convexity} \Rightarrow \text{strict convexity} \Rightarrow \text{convexity}.$$

Is the converse of either implication is true? Give a proof if it is true and give a counter example if not.

(b) Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R}^N \to \mathbb{R}$ be convex. Is their composition $f \circ g$ convex? Give a proof if so and give a counter example otherwise.

3. Prove that the following functions are convex:

(a) $|x|^\alpha, \alpha \geq 1, x \in \mathbb{R}$; (b) $\sum_{i=1}^N -\ln(1 + x_i),\ x_i \geq -1$;

(c) $x^T Px, x \in \mathbb{R}^N$, where $P \in \mathbb{R}^{N\times N}$ is symmetric and positive semi-definite.