

Homework 2

1. (a) Give the interior and relative interior of the following sets in \mathbb{R}^3 :

$$A = \{(x, y, 0) : x^2 + y^2 \leq 1\}, \quad B = \{(x, y, z) : x^2 + y^2 + z^2 < 1\}.$$

- (b) Is it true that $T \subset S \Rightarrow \text{int}(T) \subset \text{int}(S)$? Does this result hold for relative interior? Give your reasons.

2. Assume $f_1, f_2 : \mathbb{R}^N \rightarrow \mathbb{R}$ are convex functions. Decide whether the following functions are convex. Give a proof (using definition of convexity) if it is convex and give a counter example if not.

- (a) $f_1 + f_2$; (b) $f_1 \cdot f_2$;
(c) $f_1 - f_2$; (d) $\max\{f_1, f_2\}$.

3. The epigraph of a function $f : \mathbb{R}^N \rightarrow \mathbb{R}$ is defined as

$$\text{epi}(f) = \{(x, t) \in \mathbb{R}^{N+1} : f(x) \leq t\}.$$

Prove that f is a convex function if and only if $\text{epi}(f)$ is a convex set. And then use this result to solve (d) in Question 2. (Hint: show that $\text{epi}(f_1) \cap \text{epi}(f_2) = \text{epi}(\max\{f_1, f_2\})$.)