1. Suppose $f : [0,\infty) \to [0,\infty)$ s.t.

$$(CR)\int_0^\infty f := \lim_{x\to+\infty} (R)\int_0^x f(t)\,dt \text{ exists in } \mathbb{R}$$

Show that $(CR)\int_0^\infty f = (\mathcal{L})\int_0^\infty f$

(Hint: Use sequences $x_n \to \infty$ and $f\chi_{[0,x_n]}$ together with one of our convergence theo.)

2. Show that

$$\lim_{n\to\infty}\int_1^\infty \frac{\sqrt{x}}{1+nx^3} = 0$$

(Hint: the integrands are $\leq \frac{\sqrt{x}}{x^3} \in \mathcal{L}[1,\infty)$.)

3. Use the MCT and $\int_0^\infty e^{-x}\,dx = 1$ to show that

$$\lim_{n\to\infty}\int_0^n \left(1+\frac{x}{n}\right)^n \cdot e^{-2x}\,dx = 1.$$

4. Show, by BC Theorem, that

$$\lim_{n} \int_0^{\pi/2} \cancel{\sin nx}\ \sin x \cdot \arctan(nx)\, dx = \pi/2$$

5. Show that

$$\lim_{n} \int_0^1 \frac{n x^{n-1}}{1+x}\, dx = 1/2$$

(Hint: Integration by part in Calculus and then by BCT).

6. Let $f \in ABC[a,b]$. Show that it is of bounded variation in the sense that $\exists M \in (0,\infty)$ s.t.

$$\sum_{i=1}^{n} |f(x_{i-1}) - f(x_i)| \leq M$$

for all partitions $\{a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b\}$ of $[a,b]$.