Math 4030, HW 5. Due: 27 Nov 2023

(1) Let $X : U \to S$ be a regular parametrized surface with orientation N. A parallel surface to X is the following parametrized surface: $\tilde{X} = X + aN$ where $a \in \mathbb{R}$ is a constant.

   (a) Show that
$$\tilde{X}_u \times \tilde{X}_v = (1 - 2Ha + Ka^2) X_u \times X_v$$
   where H and K denote the mean curvature and Gaussian curvature of X respectively.

   (b) Show that at the regular point of $\tilde{X}$, the Gaussian curvature is given by
$$\frac{K}{1 - 2Ha + Ka^2}$$
   and the mean curvature is given by
$$\frac{H - Ka}{1 - 2Ha + Ka^2}.$$

(2) Show that there are no minimal surface in $\mathbb{R}^3$ which is closed and bounded.

(3) Let $F : U \in \mathbb{R}^2 \to \mathbb{R}^3$ be given by
$$F(u, v) = (u \sin\alpha \cos v, u \sin\alpha \sin v, u \cos\alpha)$$
where α is a constant and $U = \{(u, v) : u > 0\}$.

   (a) Show that F is a local diffeomorphism of U onto a cone C with the vertex at the origin and 2α as the angle of the vertex.

   (b) Is F a local isometry?

(4) Show that if $X : U \to S$ is an isothermal parametrization, i.e. $E = G = \lambda(u, v)$ and $F = 0$, then
$$K = -\frac{1}{2\lambda} \Delta \log \lambda.$$