Math 4030, HW 2. Due: 16 Oct 2023

(1) Show that $\{(x, y, z) : x^2 + y^2 = 1\}$ is a regular surface.
(2) Suppose $f : \mathbb{R}^3 \to \mathbb{R}$ is a differentiable function. If 0 is not a regular value of f, can we conclude that $f^{-1}(0)$ is not a regular surface? Explain your answer.
(3) Suppose $f : \mathbb{S}^2 \to \mathbb{S}^2$ is a map given by $f(x, y, z) = -(x, y, z)$. Show that f is a diffeomorphism.
(4) Show that the paraboloid $z = x^2 + y^2$ is diffeomorphic to $\mathbb{R}^2$.
(5) Suppose S is a regular surface given by the graph of a differentiable function $z = f(x, y)$. Let $p = (x_0, y_0, f(x_0, y_0)) \in S$, show that the tangent plane of S at p is given

$$z = f(x_0, y_0) + f_x(x_0, y_0) \cdot (x - x_0) + f_y(x_0, y_0) \cdot (y - y_0).$$

Moreover, show that the tangent plane is the graph of the differential df_q where $q = (x_0, y_0)$.