MATH3290 Mathematical Modeling 2023/2024
Assignment 2
Suggested Solutions

1. Let x_1, x_2, x_3 denote the number of advertisements on television, newspaper and radio respectively. We want to maximise the function $z = 100000x_1 + 40000x_2 + 18000x_3$, which is equivalent to maximising $z = 50x_1 + 20x_2 + 9x_3$. The maximisation is subject to the conditions

$$\begin{aligned} 2000x_1 + 600x_2 + 300x_3 &\leq 18200 \\ x_2 &\leq 10 \\ x_3 &\leq x_1 + x_2 \\ 10x_1 &\geq x_1 + x_2 + x_3 \end{aligned}$$

We note that the first condition can be rewritten as $20x_2 + 6x_2 + 3x_3 \leq 182$, the third condition is a result having no more than half the advertisements be from the radio, and the fourth condition is a result of having at least 10% of advertisements be on television, and can be rewritten as $-9x_1 + x_2 + x_3 \leq 0$. With this, we introduce the slack variables y_1, y_2, y_3, y_4 and solve via the simplex method in tableau format. The bolded entries help indicate the entering and exiting variables at each step.

x_1	x_2	x_3	y_1	y_2	y_3	y_4	z	RHS
0	1	0	1	0	0	0	0	10
-1	-1	1	0	1	0	0	0	0
-9	1	1	0	0	1	0	0	0
20	6	3	0	0	0	0	1	182
-50	-20	-9	0	0	0	0	1	0

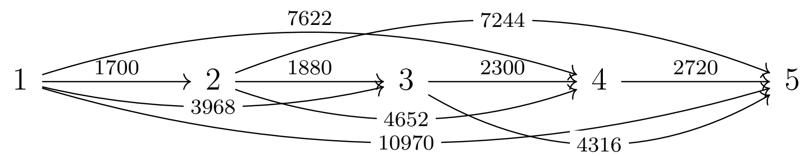
x_1	x_2	x_3	y_1	y_2	y_3	y_4	z	RHS
0	**1**	0	1	0	0	0	0	10
0	-0.7	1.15	0	1	0	0.05	0	9.1
0	3.7	2.35	0	0	1	0.45	0	81.9
1	0.3	0.15	0	0	0	0.05	0	9.1
0	-5	-1.5	0	0	0	2.5	1	455

x_1	x_2	x_3	y_1	y_2	y_3	y_4	z	RHS
0	1	0	1	0	0	0	0	10
0	0	**1.15**	0.7	1	0	0.05	0	16.1
0	0	2.35	-3.7	0	1	0.45	0	44.9
1	0	0.15	-0.3	0	0	1	0	6.1
0	0	-1.5	5	0	0	2.5	1	505

x_1	x_2	x_3	y_1	y_2	y_3	y_4	z	RHS
0	1	0	1	0	0	0	0	10
0	0	1	0.6087	0.8696	0	0.0435	0	14
0	0	0	-5.1304	-2.0435	1	0.3478	0	12
1	0	0	-0.3913	-0.1304	0	0.0435	0	4
0	0	0	5.913	1.3043	0	2.5632	1	526

It follows that the company should have 4 advertisements on television, 10 advertisements on newspaper, and 14 advertisements on radio.

2. We can formulate the problem as the following graph:



a. We compute as follows:

$$\tilde{L} = ([0^*, -], [\infty, -], [\infty, -], [\infty, -], [\infty, -])$$
$$\tilde{L} = ([0^*, -], [1700, 1], [3968, 1], [7622, 1], [10970, 1])$$

The only node to reach vertex 2 is from vertex 1, hence we have

$$\tilde{L} = ([0^*, -], [1700^*, 1], [3968, 1], [7622, 1], [10970, 1])$$

Next $L(3) = \min\{1700 + 1880, 3968\} = 1700 + 1880$, so we have

$$\tilde{L} = ([0^*, -], [1700^*, 1], [3580^*, 2], [7622, 1], [10970, 1])$$

Next $L(4) = \min\{3580 + 2300, 1700 + 4652, 7622\} = 3580 + 2300$, so we have

$$\tilde{L} = ([0^*, -], [1700^*, 1], [3580^*, 2], [5880^*, 3], [10970, 1])$$

Lastly, $L(5) = \min\{5880+2720, 3580+4316, 1700+7244, 10970\} = 3580+4316$, so we have

$$\tilde{L} = ([0^*, -], [1700^*, 1], [3580^*, 2], [5880^*, 3], [7896^*, 3])$$

Hence, the minimum cost is \$7896 and the optimal path is $1 \to 2 \to 3 \to 5$, i.e. purchase just enough stock for the first two months, then order the stock for the last two months altogether in the third month.

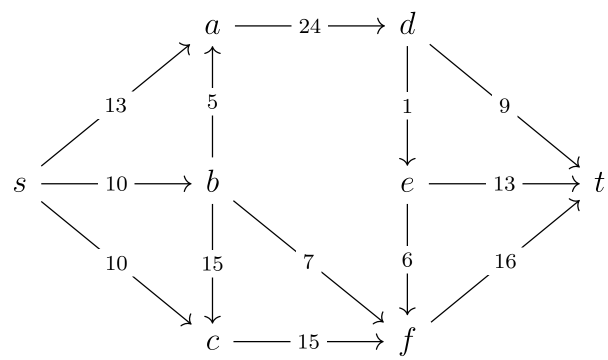
b. Clearly, at each stage, there is only one vertex. We can clearly see that compute $f_5(5) = 0, f_4(4) = 2720$, where $f_i(v)$ is the shortest distance from vertex v to the target vertex at stage i. Next, $f_3(3) = \min\{2300+f_4(4), 4316\} = 4316$ $(3 \to 5)$. We continue as follows:

$$f_2(2) = \min\{1880+f_3(3), 4652+f_4(4), 7244\} = 1880+f_3(3) = 6196 \ (2 \to 3 \to 5)$$

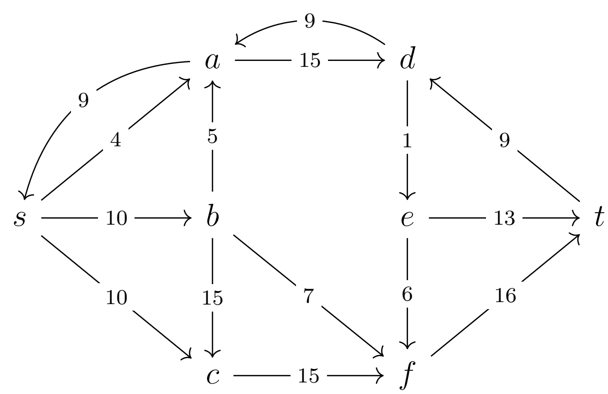
$$f_1(1) = \min\{1700+f_2(2), 3968+f_3(3), 7622+f_4(4), 10970\} = 1700+f_2(2) = 7896$$

Hence we obtain (as expected) that the lowest cost is \$7896, with path $1 \to 2 \to 3 \to 5$.

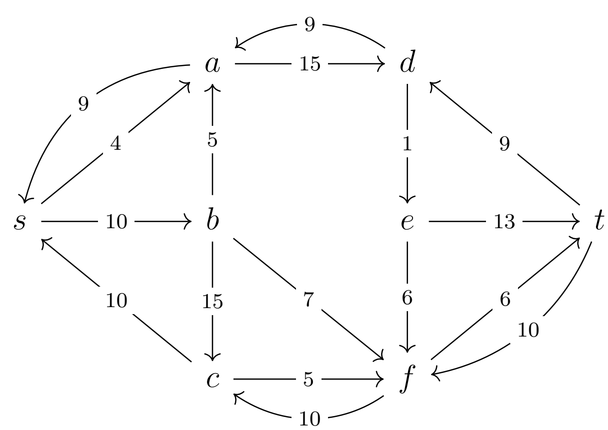
3. We first label the diagram in the following way:



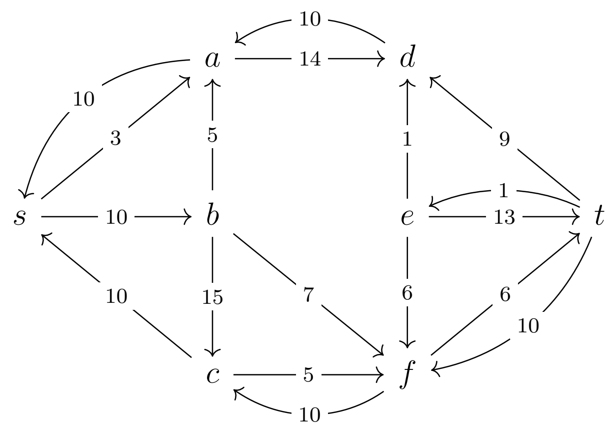
We arbitrarily choose the path $s - a - d - t$, the flow is 9, and we get that



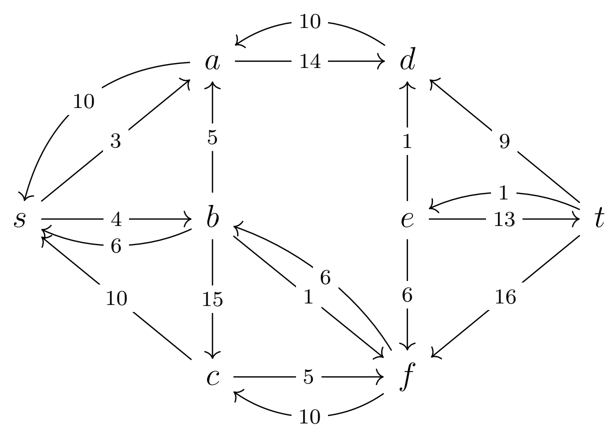
Next choose $s - c - f - t$, with flow 10.



Next choose $s - a - d - e - t$, with flow 1.



Next choose $s - b - f - t$, with flow 6.



It is clear that no more flow is possible and that the maximum flow is 26.