MATH3290 Mathematical Modeling 2023/2024
Assignment 1
Due Date: 5pm, February 20th

Note: Submit your assignment via Blackboard. Late submissions are not allowed.

1. Consider the data sets in Table 1

x	1	2	3	4	5	6	7
y_1	7	15	33	61	99	147	205
y_2	4.5	20	90	403	1,808	8,130	36,316

Table 1: Data set for Problem 1.

   (a) For (x, y_1), construct a divided difference table. What conclusions can you make about y_1? Would you use a low-order polynomial as an empirical model? If so, what order?

   (b) For (x, y_2), construct a divided difference table. Would you use a low-order polynomial as an empirical model? If not, give the reason.

2. The following data were obtained for the growth of a sheep population introduced into a new environment on the island of Tasmania.

t(year)	1814	1824	1834	1844	1854	1864
p(population)	125	275	830	1200	1750	1650

Table 2: Data set for Problem 2.

   (a) Plot the change in population versus year. Is there a trend?

   (b) Formulate a discrete dynamical system model. Use the least-squares criterion to find the model parameter.

   (c) Predict the sheep population in the year 1869.

3. **(Markov process)** A certain protein molecule can have three configurations which are denoted as C_1, C_2 and C_3. Every second, a protein molecule can make a transition from one configuration to another configuration with the following probabilities:

$$
\begin{array}{lll}
P(C_1 \to C_1) = 0.3 & P(C_1 \to C_2) = 0.2 & P(C_1 \to C_3) = 0.5 \\
P(C_2 \to C_1) = 0.3 & P(C_2 \to C_2) = 0.5 & P(C_2 \to C_3) = 0.2 \\
P(C_3 \to C_1) = 0.4 & P(C_3 \to C_2) = 0.2 & P(C_3 \to C_1) = 0.4
\end{array}
$$

   The configuration transition are demonstrated in Figure 1. (For example, the molecule will transit from C_1 to C_2 with probability 0.2.)

   Consider a living body with a fixed number of protein molecules. We let C_i^n $(i = 1, 2, 3; n = 0, 1, 2, \dots)$ be the percentage of molecules that are in configuration C_i $(i = 1, 2, 3)$ at the end of the n-th second.

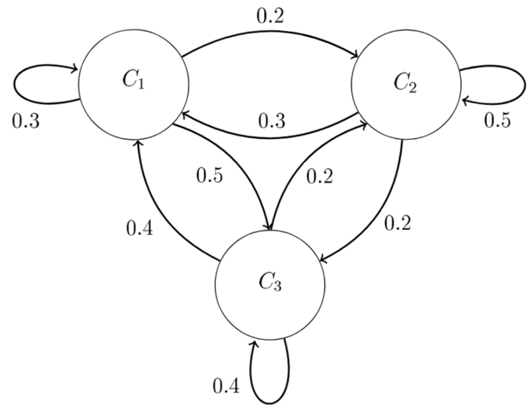

Figure 1: An illustration of the configuration transition process for Problem 3.

(a) Formulate a model for C_i^n using a system of difference equations.

(b) Find the equilibrium point, and determine its stability.

(c) Consider the three initial conditions in Table 3. Compute C_1^5, C_2^5, C_3^5 for each case. Does the long term behaviour sensitive to the initial condition?

Percentage	C_1^0	C_2^0	C_3^0
Case A	0	0	1
Case B	0	0.5	0.5
Case C	0.2	0.2	0.6

Table 3: Data set for Problem 3.