Math 3280 A 23-09-14

Review

- (Continuity of Probability)

$$P\left(\bigcup_{n=1}^{\infty} E_n\right) = \lim_{n\to\infty} P(E_n) \quad \text{if } (E_n) \text{ is increasing}$$

$$P\left(\bigcap_{n=1}^{\infty} E_n\right) = \lim_{n\to\infty} P(E_n) \quad \text{if } (E_n) \text{ is decreasing.}$$

§ 2.6 Sample space having equally likely outcomes.

In many experiments, it is natural to assume that all outcomes have the same chance to occur.

In this case,

$$P(E) = \frac{\#\text{ of outcomes in } E}{\#\text{ of outcomes in } S} = \frac{\# E}{\# S}.$$

Example 1. If two dices are rolled, what is the prob. that the sum of two outcomes is equal to 8 ?

Solution: Let E be the event that the sum of two outcomes is equal to 6. Then

$$E = \{ (i,j) : \quad i, j \in \{1, 2, \cdots, 6\}, \ i+j=8 \}$$

$$= \{ (2,6), (3,5), (4,4), (5,3), (6,2) \}$$

and

$$S = \{ (i,j) : \quad i, j \in \{1, 2, \cdots, 6\} \}$$

Hence $P(E) = \dfrac{\# E}{\# S} = \dfrac{5}{36}$. $\blacksquare$

Exer 2.

A committee of 5 is to be selected from a group of 6 men and 9 women. If the
selection is made randomly, what is the probability that the committee consists of 3
men and 2 women?

Solution: Let E denote the event that the selected committee consists of 3 men and 2 women.

Let S be whole sample space.

Then

$$\# S = \binom{15}{5},$$

$$\# E = \binom{6}{3} \cdot \binom{9}{2}.$$

Hence $P(E) = \frac{\# E}{\# S} = \frac{\binom{6}{3}\binom{9}{2}}{\binom{15}{5}}.$

$$\binom{n}{m} = \frac{n!}{m!(n-m)!}. \quad \left(n! = n \times (n-1) \times \cdots 1 \quad 0! = 1 \right)$$

$\boxed{//}$

Exer 3.

In the game of bridge, the entire deck of 52 cards is dealt out to 4 players. What is the probability that
(a) one of the players receives all 13 spades;
(b) each player receives 1 ace?

(a)

Solution: Let E be the event that one of the players receives all 13 spades.

Let E_i be the event that the i-th player receives all 13 spades, $i=1,2,3,4$.

$E = \bigcup_{i=1}^{4} E_i$, $E_1, \cdots, E_4$ are mutually exclusive.

So $P(E) = P(E_1) + P(E_2) + P(E_3) + P(E_4)$.

$$\# E_1 = \binom{39}{13} \cdot \binom{26}{13} \cdot \binom{13}{13}$$

Similarly, $\# E_2 = \# E_3 = \# E_4 = \# E_1$.

$$\# S = \binom{52}{13}\binom{39}{13}\binom{26}{13}\binom{13}{13}$$

Hence

$$P(E_1) = \frac{\# E_1}{\# S} = \frac{\binom{39}{13}\binom{26}{13}}{\binom{52}{13}\binom{39}{13}\binom{26}{13}} = \frac{1}{\binom{52}{13}}$$

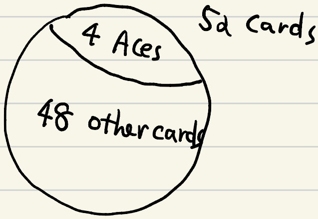
So is $P(E_i)$, $i=2,3,4$.

$$P(E) = \sum_{i=1}^{4} P(E_i) = \frac{4}{\binom{52}{13}}.$$

(b)

Let F be the event that each player receives an Ace.

$$\#F = \binom{4}{1}\cdot\binom{48}{12}\cdot\binom{3}{1}\cdot\binom{36}{12}\cdot\binom{2}{1}\binom{24}{12}$$



Hence $P(F) = \dfrac{\binom{4}{1}\binom{48}{12}\binom{3}{1}\binom{36}{12}\binom{2}{1}\binom{24}{12}}{\binom{52}{13}\binom{39}{13}\binom{26}{13}}$.

Exer 4. A deck of 52 cards in dealt out. What is the probability that the first ace occurs in the 14th card.

Solution: Let E denote the event that the first ace occurs in the 14 th card. Let S denote the sample space.

Then $\# S = 52!$

$$\# E = 48 \times 47 \times \cdots \times 36 \times 4 \times (38!)$$

Hence

$$P(E) = \frac{\# E}{\# S} = \frac{48 \times 47 \times \cdots \times 36 \times 4}{52 \times 51 \times \cdots \times 39}.$$

Chap 3. Conditional probability and independence

§3.1 Conditional probability.

Example 1: Let us roll two dices. Suppose the first die is a 3. Given this information, what is the prob. that the sum of 2 dices equals 8?

Sol: F — the event that the first die is 3

E — the event that the sum of 2 dices equals 8.

$$F = \{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)\}$$

$$E = \{(i,j) \in \{1,2,3,4,5,6\}^2 : \ i+j = 8\}.$$

Prob of each outcome in F is $\frac{1}{6}$.

Hence the (conditional) prob of E given F is $\frac{1}{6}$.

Def. (Conditional prob.).

Let E, F be two events for a random experiment. Suppose $P(F) > 0$. Then the conditional prob. of E given F is

$$P(E|F) = \frac{P(EF)}{P(F)}.$$

Example 2: A coin is flipped twice. What is the conditional prob. that both flips land on heads given that the first flip lands on head?

Sol: Let F be the event that the first flip lands on head. That

$$F = \{(H, H), (H, T)\}.$$

Let E be the event that both flips land on heads.

$$E = \{(H, H)\}.$$

By def, $$P(E|F) = \frac{P(EF)}{P(F)} = \frac{P\{(H,H)\}}{P\{(H,H), (H,T)\}}$$

Notice that $S = \{(H,H), (H,T), (T,H), (T,T)\}$

Prop. (Multiplicative rule)

- $P(E_1 E_2) = P(E_1)\, P(E_2|E_1)$
- $P(E_1 E_2 \cdots E_n)$
$$= P(E_1) \cdot P(E_2|E_1) \cdot P(E_3|E_1E_2) \cdots \cdot P(E_n|E_1E_2\cdots E_{n-1})$$

Pf. Since $P(E_2|E_1) = \dfrac{P(E_1E_2)}{P(E_1)}$, so

$$P(E_1E_2) = P(E_1)\cdot P(E_2|E_1).$$

To see the second identity,

$$\text{RHS} = P(E_1) \cdot \frac{P(E_1E_2)}{P(E_1)} \cdot \frac{P(E_1E_2E_3)}{P(E_1E_2)} \cdots \frac{P(E_1\cdots E_n)}{P(E_1\cdots E_{n-1})}$$

$$= P(E_1\cdots E_n). \quad \blacksquare$$