THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH3280A Introductory Probability 2023-2024 Term 1
Homework Assignment 2
Due Date: 3 October, 2023 (Tuesday)

I declare that the assignment here submitted is original except for source material explicitly acknowledged, the piece of work, or a part of the piece of work has not been submitted for more than one purpose (i.e. to satisfy the requirements in two different courses) without declaration, and that the submitted soft copy with details listed in the "Submission Details" is identical to the hard copy, if any, which has been submitted. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained on the University website https://www.cuhk.edu.hk/policy/academichonesty/

It is also understood that assignments without a properly signed declaration by the student concerned will not be graded by the course teacher.

____________________ ____________________
Signature Date

General Regulations

- All assignments will be submitted and graded on Blackboard. You can view your grades and submit regrade requests here as well.
- Late assignments will receive a grade of 0.
- Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your assignments). Please write your answers using a black or blue pen, NOT any other color or a pencil.
- Write your solutions on A4 white paper. Please do not use any colored paper and make sure that your written solutions are a suitable size (easily read). Failure to comply with these instructions will result in a 10-point deduction.
- Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your answers getting good marks on this homework. Neatness and organization are also essential.

1. If two fair dice are rolled, what is the conditional probability that the first one lands on 6 given that the sum of the dice is i? Compute for all values of i between 2 and 12.

2. What is the probability that at least one of a pair of fair dice lands on 6, given that the sum of the dice is i, $i = 2, 3, \cdots, 12$?

3. Two cards are randomly chosen without replacement from an ordinary deck of 52 cards. Let B be the event that both cards are aces, let A_h be the event that the ace of hearts is chosen, and let A be the event that at least one ace is chosen. Find

   (a) $P(B|A_h)$

   (b) $P(B|A)$

4. Urn I contains 2 blue and 4 white balls, whereas urn II contains 1 blue and 1 white ball. A ball is randomly chosen from urn I and put into urn II, and a ball is then randomly selected from urn II. What is

   (a) the probability that the ball selected from urn II is blue?

   (b) the conditional probability that the transferred ball was blue given that a blue ball is selected from urn II?

5. E and F play a series of games. Each game is independently won by E with probability p and by F with probability $1 - p$. They stop when the total number of wins of one of the players is two greater than that of the other player. The player with the greater number of total wins is declared the winner of the series.

   (a) Find the probability that a total of 4 games are played.

   (b) Find the probability that E is the winner of the series.

6. Let $A \subset B$. Express the following probabilities as simply as possible:

   (a) $P(A|B)$;

   (b) $P(A|B^c)$;

   (c) $P(B|A)$;

   (d) $P(B|A^c)$.

7. Independent trials that result in a success with probability q and a failure with probability $1 - q$ are called Bernoulli trials. Let Q_n denote the probability that n Bernoulli trials result in an even number of successes (0 being considered an even number). Show that

$$Q_n = q(1 - Q_{n-1}) + (1 - q)Q_{n-1},\ n \geq 1$$

   and use this formula to prove (by induction) that

$$Q_n = \frac{1 + (1 - 2q)^n}{2}$$

8. Let C_n denote the probability that no run of 3 consecutive heads appears in n tosses of a fair coin. Show that

$$C_n = \frac{1}{2}C_{n-1} + \frac{1}{4}C_{n-2} + \frac{1}{8}C_{n-3}$$

$$C_0 = C_1 = C_2 = 1$$

Find C_8. (*Hint:* condition on the first trial)

9. For a fixed event B, show that the collection $P(A|B)$, defined for all events A, satisfies the three axioms for a probability. Furthermore prove that

$$P(A|B) = P(A|B \cap C)P(C|B) + P(A|B \cap C^c)P(C^c|B)$$