THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH3093, Assignment 1

Date Due: Jan 17, 2024

(1) Verify that $f(x) = e^{inx}$ $(n \in \mathbb{Z})$ is periodic with period 2π and that

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{inx}\, dx = \begin{cases} 1 & \text{if } n = 0, \\ 0 & \text{if } n \neq 0. \end{cases}$$

Using this fact to show that if $n, m \geq 1$ $(n, m \in \mathbb{Z})$ we have

$$\frac{1}{\pi}\int_{-\pi}^{\pi} \cos nx\ \cos mx\, dx = \begin{cases} 0 & \text{if } n \neq m, \\ 1 & \text{if } n = m, \end{cases}$$

and similarly

$$\frac{1}{\pi}\int_{-\pi}^{\pi} \sin nx\ \sin mx\, dx = \begin{cases} 0 & \text{if } n \neq m, \\ 1 & \text{if } n = m. \end{cases}$$

Finally, show that

$$\frac{1}{\pi}\int_{-\pi}^{\pi} \sin nx\ \cos mx\, dx = 0 \quad \text{for all } n, m \in \mathbb{Z}.$$

(2) Consider the Laplacian $\triangle u \equiv \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ on the disc $\{(x, y) :\ x^2 + y^2 < 1\}$.

(a) Show that in polar coordinates $U(r, \theta) = u(x, y)$ satisfies

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} + \frac{1}{r^2}\frac{\partial^2 U}{\partial \theta^2} = 0 \tag{0.1}$$

in $\{(r, \theta) :\ 0 \leq r < 1,\ \theta \in \mathbb{R}\}$.

(b) Show that all solutions of the form $F(r)G(\theta)$ to (0.1) are one of the forms $r^n(A_n \cos n\theta + B_n \sin n\theta)$, $n = 0, 1, 2, \ldots$.