

**THE CHINESE UNIVERSITY OF HONG KONG**  
**Department of Mathematics**  
**MATH 3030 Abstract Algebra 2023-24**  
**Tutorial 5**  
**12th October 2023**

- Tutorial exercise would be uploaded to blackboard on Tuesdays provided that there is a tutorial class on that Thursday. You are not required to hand in the solution, but you are advised to try the problems before tutorial classes.
  - Please send an email to [echlam@math.cuhk.edu.hk](mailto:echlam@math.cuhk.edu.hk) if you have any questions.
1. Suppose that  $N \triangleleft G$  and  $N \cap G' = \{e\}$  where  $G'$  is the commutator subgroup of  $G$ , show that  $N \leq Z(G)$ .
  2. (a) Let  $H, K$  be normal subgroups of  $G$ , define  $\phi : G/(H \cap K) \rightarrow G/H \times G/K$  by  $\phi(aH \cap K) = (aH, aK)$ . Prove that  $\phi$  is a well-defined and injective homomorphism.  
(b) Prove that  $\phi$  is surjective if and only if  $G = HK$ .  
(c) Prove that  $\mathbb{Z}_{pq} \cong \mathbb{Z}_p \times \mathbb{Z}_q$  for distinct prime numbers  $p, q$ .
  3. Prove that the group of upper triangular matrices  $B_2 \leq GL(2, \mathbb{C})$  is solvable.
  4. Suppose  $G$  is a finite solvable group, let  $0 \neq N \triangleleft G$  is a minimal normal subgroup, i.e. there is no proper nontrivial subgroup  $M \leq N$  so that  $M$  is normal in  $G$ , prove that  $N$  is abelian.
  5. Does  $\mathbb{Q}$  have a composition series?
  6. Find a composition series for  $D_8$  the symmetry group of regular 8-gons, and also a composition series for  $\mathbb{Z}_{48}$ .
  7. Let  $f : G \rightarrow H$  be a homomorphism, if  $G$  is solvable, show that the image  $f(G)$  is also solvable.