

MATH2058 Honours Mathematical Analysis I

Tutorial 8

In the tutorial, we discuss the questions in Home Test and the following.

Theorem 8.8. *If f is a continuous function defined on a compact set A , then f is a bounded function. Moreover, there are x_1 and x_2 in A such that $f(x_1) = \min\{f(x) : x \in A\}$ and $f(x_2) = \max\{f(x) : x \in A\}$.*

Theorem 8.10. *If f is a continuous function defined on a compact set A , then the image $f(A) := \{f(x) : x \in A\}$ is compact.*

Example 1. Suppose that $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous on \mathbb{R} and that $\lim_{x \rightarrow -\infty} f = 0$ and $\lim_{x \rightarrow \infty} f = 0$.

- (a) Prove that f attains either a maximum or minimum on \mathbb{R} .
- (b) Give an example to show that both a maximum and a minimum need not be attained.