

MATH2058 Honours Mathematical Analysis I  
Tutorial 10

**Uniform Continuity**

**Uniform Continuity Theorem.** *Let  $f$  be a continuous function on  $A$ . If  $A$  is compact, then  $f$  is uniformly continuous on  $A$ .*

**Example 1.** Let  $f$  and  $g$  be uniformly continuous functions on  $A \subseteq \mathbb{R}$ .

- (a) Is it always true that their product  $fg$  is also uniformly continuous on  $A$ ?
- (b) What if we further assume that  $f$  and  $g$  are both bounded on  $A$ ?

**Example 2.** A function  $f : \mathbb{R} \rightarrow \mathbb{R}$  is said to be periodic on  $\mathbb{R}$  if there exists a number  $p > 0$  such that  $f(x + p) = f(x)$  for all  $x \in \mathbb{R}$ . Prove that a continuous periodic function on  $\mathbb{R}$  is bounded and uniformly continuous on  $\mathbb{R}$ .

**Example 3.** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a continuous function. Suppose  $\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow \infty} f(x) = \ell \in \mathbb{R}$ . Show that  $f$  is uniformly continuous on  $\mathbb{R}$ .