MATH2058: Mathematical Analysis I
Home Test 1 **5:00pm, 27 Oct 2023**

Important Notice:

♣ The answer paper **Must be submitted before 28 Oct 2023 at 5:00 pm.**
¶ You are NOT allowed to resubmit your answer paper again after submission
♠ The answer paper MUST BE sent to the CU Blackboard.
♠ The answer paper MUST BE sent in pdf format IN ONE-file (Other format files, for example, jpg files, are NOT ACCEPTED).
✠ The answer paper **Must include your name and student ID in each page.**

Answer ALL Questions

1. **(10 points)**
   Let C be a countably infinite set of non-negative real valued functions defined on $\mathbb{R}$. Assume that for any sequence (g_m) in C and for any sequence of real numbers (a_m), we have
   $$\sup\{\sum_{m=1}^{r} g_m(a_m) : r = 1, 2...\} < \infty.$$
   Show that $\lim_{m\to\infty} \sup\{g(x_m) : g \in C\} = 0$ for all sequences (x_m).

2. **(10 points)**
   For each $n = 1, 2, ...,$ let $f_n(x) := \sin^n x$, $x \in \mathbb{R}$. Show that there is a subsequence (f_{n_k}) of (f_n) such that $\lim_{k\to\infty} f_{n_k}(r)$ exists for any rational number r.

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3. **(20 points)**

Let $f_n : [0,1] \to [0,\infty)$ be a sequence of functions, $n = 1, 2, ...$ Assume that for each $n = 1, 2, ...,$, we have

$$\sup\{\sum_{t\in F} f_n(t) : F \text{ is any finite subset of } [0,1]\} < \infty.$$

(a) For each $n = 1, 2..$, let $D_n := \{t \in [0,1] : f_n(t) = 0\}$. Show that $D := \bigcap_{n=1}^{\infty} D_n \neq \emptyset$.

(b) Does there exist a limit point of the set D defined in above?

$***$ **END OF PAPER** $***$