MATH2050C Assignment 3

Deadline: Jan 30, 2024.

Hand in: Section 3.1 no. 5d, 6d, 8, 12, 14, 18. Section 3.2 no 13.

Section 3.1 no. 2, 3, 5, 6, 7, 8, 12, 14, 16, 17, 18. **Section 3.1** no. 6ad, 11, 13.

This is basic stuff. You are strongly advised to do all exercises in these sections unless you feel confident after working out some of them.

Supplementary Problems

1. Find the limit of $\{x_n\}, x_n = \frac{7n^2+3}{n^2-n-5}$. Determine n_0 explicitly for given $\varepsilon > 0$. Recall definition: $\{x_n\}$ converges to x if for each $\varepsilon > 0$, there is some n_0 such that $|x_n - x| < \varepsilon$ for all $n \geq n_0$.

2. Let $p(x) = a_0 + a_1 x + \cdots + a_n x^n, a_n \neq 0$, and $q(x) = b_0 + b_1 x + \cdots + b_m x^m, b_m \neq 0$, be two polynomials. Consider the sequence $x_k = p(k)/q(k), k \geq 1$, (when k is large, $q(k)$ does not vanish, so you may assume that q is always non-zero). Prove that

   (a) When $n = m$, $\lim_{k\to\infty} x_k = a_n/b_m$;

   (b) When $n > m$, $\{x_k\}$ is divergent ; and

   (c) When $n < m$, $\lim_{k\to\infty} x_k = 0$.

3. Suppose that $x_n \to x, x_n \geq 0$. Show that $x_n^{p/q} \to x^{p/q}$ for $p, q \in \mathbb{N}$.