MATH 2050 Mathematical Analysis I

Pre-requisites: MATH 1050/1058 (and MATH 1010/1018)

- Set theoretic concepts ($\forall, \exists, \in, \subseteq, \cap, \cup$)
- Number systems ($\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$)
- Functions $f: A \to B$
- ★ Proof Writing

(Reference: Bartle Chapter 1)

Thm: $\nexists\, r \in \mathbb{Q}$ s.t. $r^2 = 2$. [i.e. $\sqrt{2}$ is irrational.]

Proof: We will prove "by contradiction".

Suppose NOT. Then, $\exists\, r \in \mathbb{Q}$ s.t. $r^2 = 2$.

Since $r \in \mathbb{Q}$, we can find $p, q \in \mathbb{Z}$, $q \neq 0$ s.t.

$$r = \frac{p}{q}$$ where p, q are "relatively prime".

- As $2 = r^2 = \left(\frac{p}{q}\right)^2 = \frac{p^2}{q^2} \Rightarrow p^2 = 2q^2$ (#)

  i.e. p^2 is even $\Rightarrow$ p is even, i.e. $\exists\, k \in \mathbb{Z}$ s.t. $p = 2k$.

- Plug $p = 2k$ into (#).

$$4k^2 = p^2 = 2q^2 \Rightarrow q^2 = 2k^2 \quad (\#\#)$$

Similar argument $\Rightarrow$ q^2 is even $\Rightarrow$ q is even

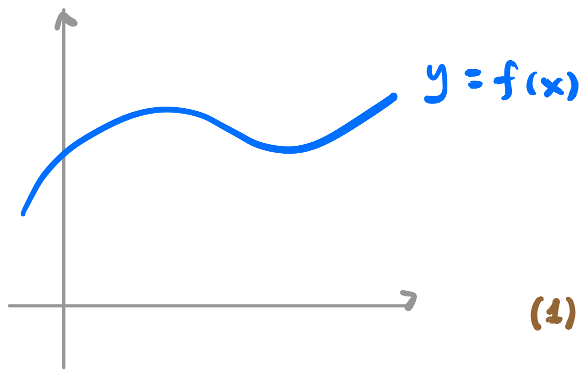
Thus, both p & q are even, which contradicts the fact that they are relatively prime.

$\blacksquare$

An Overview of MATH 2050 (and 2060 / 3060)

Goal: Study the "analytic properties" of functions $f: \mathbb{R} \to \mathbb{R}$

[e.g. $\underbrace{\text{limit, continuity}}_{\text{MATH 2050}}$, $\underbrace{\text{differentiable / integrable}}_{\text{MATH 2060 (3060)}}$?]



Q: $\exists\, f: \mathbb{R} \to \mathbb{R}$ s.t. continuous everywhere but nowhere differentiable ?

Summary (MATH 2050)

(1) [Ch. 2] $\mathbb{R}$ as complete ordered field.

(2) [Ch. 3] limit of sequences $\lim (x_n)$

(3) [Ch. 4] limit of functions $\lim\limits_{x \to a} f(x)$

(4) [Ch. 5] continuity of functions