Math 2050, HW 1. Due: 25 Sep 2022

(1) Using the Axioms to show that for all $a, b \in \mathbb{R}$,

$$(-a)^2 = a^2 \quad \text{and} \quad (a + (-b))^2 = a^2 + (-2ab) + b^2.$$

(2) Show that for all $n \in \mathbb{N}$,

$$(n, n+1) \cap \mathbb{N} = \emptyset.$$

Show further that if $m, n \in \mathbb{Z}$ such that $m < n$, then $m+1 \leq n$.

(3) Suppose S is a non-empty bounded subset in $\mathbb{R}$. Is $\sup S$ necessarily inside S? Justify your answer.

(4) Show that if A, B are bounded subsets of $\mathbb{R}$. Show that

$$\sup(A+B) = \sup A + \sup B, \quad \text{and} \quad \inf(A+B) = \inf A + \inf B$$

where $A + B = \{a + b : a \in A, \quad b \in B\}$.

(5) Show that $2^n \geq 1 + n$ for all $n \in \mathbb{N}$. Show further that for any $x > 0$, there is $n \in \mathbb{N}$ such that $\frac{1}{2^n} < x$.

(6) Show by using completeness that there is $x \in \mathbb{R} \setminus \mathbb{Q}$ so that $x > 0$ and $x^3 = 2$.