Math 2050, HW 3, Due:23 Oct

(1) Establish the convergence or the divergence of the sequence $\{x_n\}_{n=1}^{\infty}$ where

$$x_n = \sum_{k=1}^{n} \frac{1}{n+k}$$

(2) Prove that the sequence $\{x_n\}_{n=1}^{\infty}$ where

$$x_n = \sum_{k=1}^{n} \frac{1}{k^2}$$

is convergence using the monotone convergence Theorem.

(3) Suppose $x_n \geq 0$ for all $n \in \mathbb{N}$ and $\lim_{n\to+\infty}(-1)^n x_n$ exists. Show that $\{x_n\}_{n=1}^{\infty}$ is convergent.

(4) Show that if $\{x_n\}_{n=1}^{\infty}$ is unbounded, then there exists a sub-sequence $\{x_{n_j}\}_{j=1}^{\infty}$ which is non-zero so that $\frac{1}{x_{n_j}} \to 0$ as $j \to +\infty$.

(5) Suppose every sub-sequence of $\{x_n\}_{n=1}^{\infty}$, there exists a sub-sequence that converges to 0, show that $\{x_n\}_{n=1}^{\infty}$ is convergent with limit 0.