Math 2050, HW 2

(1) If S is a non-empty subset of $\mathbb{R}$. Show that S is bounded if and only if there exists a closed and bounded interval I such that $S \subseteq I$.

(2) Let $f, g : S \to \mathbb{R}$ be two real-valued functions. Suppose that $\sup\{f(x) + g(x) : x \in S\}$, $\sup\{f(x) : x \in S\}$ and $\sup\{g(x) : x \in S\}$ exist in $\mathbb{R}$. Show that

$$\sup\{f(x) + g(x) : x \in S\} \leq \sup\{f(x) : x \in S\} + \sup\{g(x) : x \in S\}.$$

Show that in general $\leq$ cannot be replaced by $=$ by providing counter-example.

(3) (a) Let $x_1 = 1$ and $x_{n+1} = x_n + \frac{1}{x_n}$ for all $n \in \mathbb{N}$. Determine whether $\{x_n\}$ is convergent or not.

(b) Let $x_1 = 1$ and $x_{n+1} = \sqrt{2x_n}$ for all $n \in \mathbb{N}$, show that $\{x_n\}$ is convergent.

(4) (a) Show that $\{\dfrac{n+3}{n^3 - 2n + 4}\}_{n=1}^{\infty}$ is convergent.

(b) Show that $\{(5n^6)^{\frac{1}{n}}\}_{n=1}^{\infty}$ is convergent.