MATH 2020B Advanced Calculus II
2023-24 Term 2
Suggested Solution of Homework 1

Refer to Textbook: Thomas' Calculus, Early Transcendentals, 13th Edition

Exercise 15.1

7. Evaluate the iterated integral $\int_0^1 \int_0^1 \frac{y}{1+xy}\,dx\,dy$.

Solution. $\int_0^1 \int_0^1 \frac{y}{1+xy}\,dx\,dy = \int_0^1 [\ln|1+xy|]_0^1\,dy = \int_0^1 \ln|1+y|\,dy$
$= [y\ln|1+y| - y + \ln|1+y|]_0^1 = 2\ln 2 - 1.$ ◀

14. Evaluate the iterated integral $\int_{-1}^2 \int_1^2 x\ln y\,dy\,dx$.

Solution. $\int_{-1}^2 \int_1^2 x\ln y\,dy\,dx = \int_{-1}^2 x\,[y\ln|y| - y]_1^2\,dx = \int_{-1}^2 (2\ln 2 - 1)x\,dx$
$= (2\ln 2 - 1)\left[\frac{x^2}{2}\right]_{-1}^2 = 3\ln 2 - \frac{3}{2}.$ ◀

22. Evaluate the double integral over the given region R.

$$\iint_R \frac{y}{x^2y^2+1}\,dA, \quad R:\ 0 \le x \le 1, \quad 0 \le y \le 1.$$

Solution. $\iint_R \frac{y}{x^2y^2+1}\,dA = \int_0^1 \int_0^1 \frac{y}{(xy)^2+1}\,dx\,dy = \int_0^1 \left[\tan^{-1}(xy)\right]_0^1\,dy$
$= \int_0^1 \tan^{-1} y\,dy = \left[y\tan^{-1} y - \frac{1}{2}\ln|1+y^2|\right]_0^1 = \frac{\pi}{4} - \frac{1}{2}\ln 2.$ ◀

27. Find the volume of the region bounded above by the plane $z = 2 - x - y$ and below by the square $R: 0 \le x \le 1\ 0 \le y \le 1$.

Solution. Volume $= \iint_R (2-x-y)\,dA = \int_0^1 \int_0^1 (2-x-y)\,dy\,dx = \int_0^1 \left[2y - xy - \frac{1}{2}y^2\right]_0^1 dx$
$= \int_0^1 (\frac{3}{2} - x)\,dx = \left[\frac{3}{2}x - \frac{1}{2}x^2\right]_0^1 = 1.$ ◀

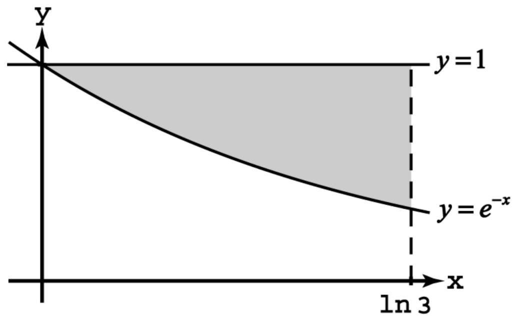
29. Find the volume of the region bounded above by the surface $z = 2\sin x\cos y$ and below by the square $R: 0 \le x \le \pi/2,\ 0 \le y \le \pi/4$.

Solution. Volume $= \iint_R 2\sin x\cos y\,dA = \int_0^{\pi/2} \int_0^{\pi/4} 2\sin x\cos y\,dy\,dx$
$= \int_0^{\pi/2} [2\sin x\sin y]_0^{\pi/4}\,dx \int_0^{\pi/2} \sqrt{2}\sin x\,dx = \left[-\sqrt{2}\cos x\right]_0^{\pi/2} = \sqrt{2}.$ ◀

Exercise 15.2

15. Write an iterated integral for $\iint_R dA$ over the described region R using (a) vertical cross-sections, (b) horizontal cross-sections.
R: bounded by $y = e^{-x}$, $y = 1$, and $x = \ln 3$.

Solution.



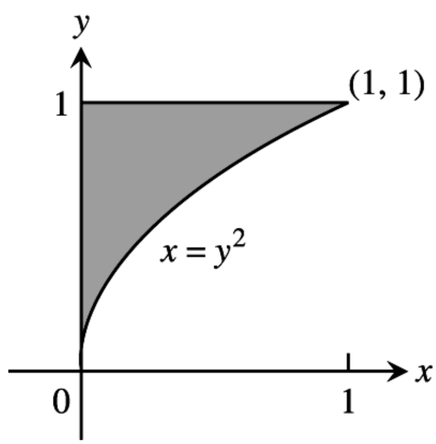
(a) $\iint_R dA = \int_0^{\ln 3} \int_{e^{-x}}^{1} dy\ dx = \ln 3 - \frac{2}{3}$.

(b) $\iint_R dA = \int_{1/3}^{1} \int_{-\ln y}^{\ln 3} dx\, dy = \ln 3 - \frac{2}{3}$.

◀

23. Sketch the region of integration and evaluate the integral: $\int_0^1 \int_0^{y^2} 3y^3 e^{xy}\, dx\, dy$.

Solution.

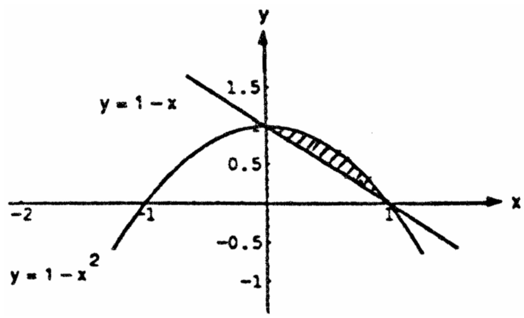


$$\int_0^1 \int_0^{y^2} 3y^3 e^{xy}\, dx\, dy = \int_0^1 \left[3y^2 e^{xy}\right]_0^{y^2} dy = \int_0^1 \left(3y^2 e^{y^3} - 3y^2\right) dy$$
$$= \left[e^{y^3} - y^3\right]_0^1 = e - 2.$$

◀

36. Sketch the region of integration and write an equivalent double integral with the order of integration reversed: $\displaystyle\int_0^1 \int_{1-x}^{1-x^2} dy\, dx.$

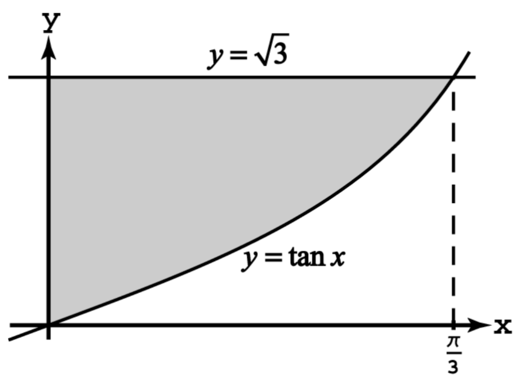
Solution.



$$\int_0^1 \int_{1-x}^{1-x^2} dy\, dx = \int_0^1 \int_{1-y}^{\sqrt{1-y}} dx\, dy. \qquad \blacktriangleleft$$

46. Sketch the region of integration and write an equivalent double integral with the order of integration reversed: $\displaystyle\int_0^{\sqrt{3}} \int_0^{\tan^{-1} y} \sqrt{xy}\, dx\, dy.$

Solution.



$$\int_0^{\sqrt{3}} \int_0^{\tan^{-1} y} \sqrt{xy}\, dx\, dy = \int_0^{\pi/3} \int_{\tan x}^{\sqrt{3}} \sqrt{xy}\, dy\, dx. \qquad \blacktriangleleft$$

62. Find the volume of the wedge cut from the first octant by the surface $z = 4 - x^2 - y$.

Solution. Volume $\displaystyle = \int_0^2 \int_0^{4-x^2} (4 - x^2 - y)\, dy\, dx = \int_0^2 \left[(4 - x^2)y - \frac{y^2}{2}\right]_0^{4-x^2} dx$

$$= \int_0^2 \frac{1}{2}(4 - x^2)^2\, dx = \int_0^2 \left(8 - 4x^2 + \frac{x^4}{2}\right) dx = \left[8x - \frac{4}{3}x^3 + \frac{1}{10}x^5\right]_0^2 = \frac{128}{15}. \qquad \blacktriangleleft$$