MATH 2020A Advanced Calculus II
2023-24 Term 1
Suggested Solution of Homework 1

Refer to Textbook: Thomas' Calculus, Early Transcendentals, 13th Edition

Exercise 15.1

2. Evaluate the iterated integral $\int_0^2 \int_{-1}^1 (x-y)\,dy\,dx$.

Solution. $\int_0^2 \int_{-1}^1 (x-y)\,dy\,dx = \int_0^2 \left[xy - \frac{1}{2}y^2\right]_{-1}^1 dx = \int_0^2 2x\,dx = \left[x^2\right]_0^2 = 4.$ ◀

6. Evaluate the iterated integral $\int_0^3 \int_{-2}^0 (x^2y-2xy)\,dy\,dx$.

Solution. $\int_0^3 \int_{-2}^0 (x^2y-2xy)\,dy\,dx = \int_0^3 \left[\frac{x^2y^2}{2} - xy^2\right]_{-2}^0 dx = \int_0^2 (4x-2x^2)\,dx$
$= \left[2x^2 - \frac{2x^3}{3}\right]_0^3 = 0.$ ◀

13. Evaluate the iterated integral $\int_1^4 \int_1^e \frac{\ln x}{xy}\,dx\,dy$.

Solution. $\int_1^4 \int_1^e \frac{\ln x}{xy}\,dx\,dy = \int_1^4 \left[\frac{1}{2y}(\ln x)^2\right]_1^e dy = \int_1^4 \frac{1}{2y}\,dy = \left[\frac{\ln y}{2}\right]_1^4 = \ln 2.$ ◀

15. Evaluate the double integral over the given region R.

$$\iint_R (6y^2-2x)\,dA, \quad R:\ 0 \le x \le 1, \quad 0 \le y \le 2.$$

Solution. $\iint_R (6y^2-2x)\,dA = \int_0^1 \int_0^2 (6y^2-2x)\,dy\,dx = \int_0^1 \left[2y^3-2xy\right]_0^2 dx$
$= \int_0^1 (16-4x)\,dx = \left[16x-2x^2\right]_0^1 = 14.$ ◀

17. Evaluate the double integral over the given region R.

$$\iint_R xy\cos y\,dA, \quad R:\ -1 \le x \le 1, \quad 0 \le y \le \pi.$$

Solution. $\iint_R xy\cos y\,dA = \int_{-1}^1 \int_0^\pi xy\cos y\,dy\,dx = \int_{-1}^1 \left[xy\sin y + x\cos y\right]_0^\pi dx$
$= \int_{-1}^1 (-2x)\,dx = \left[-x^2\right]_{-1}^1 = 0.$ ◀

21. Evaluate the double integral over the given region R.

$$\iint_R \frac{xy^3}{x^2+1}\,dA, \quad R:\ 0 \le x \le 1, \quad 0 \le y \le 2.$$

Solution. $\displaystyle\iint_R \frac{xy^3}{x^2+1}\,dA = \int_0^1 \int_0^2 \frac{xy^3}{x^2+1}\,dy\,dx = \int_0^1 \left[\frac{xy^4}{4(x^2+1)}\right]_0^2 dx = \int_0^1 \frac{4x}{x^2+1}\,dx$
$= \left[2\ln|x^2+1|\right]_0^1 = 2\ln 2.$ ◀

26. Find the volume of the region bounded above by the elliptical paraboloid $z = 16 - x^2 - y^2$ and below by the square $R: 0 \le x \le 2,\ 0 \le y \le 2$.

Solution. Volume $\displaystyle= \iint_R (16 - x^2 - y^2)\,dA = \int_0^2 \int_0^2 (16 - x^2 - y^2)\,dy\,dx$
$\displaystyle= \int_0^2 \left[16y - x^2y - \frac{y^3}{3}\right]_0^2 dx = \int_0^2 (\frac{88}{3} - 2x^2)\,dx = \left[\frac{88x}{3} - \frac{2x^3}{3}\right]_0^2 = \frac{160}{3}.$ ◀

30. Find the volume of the region bounded above by the surface $z = 4 - y^2$ and below by the rectangle $R: 0 \le x \le 1,\ 0 \le y \le 2$.

Solution. Volume $\displaystyle= \iint_R (4 - y^2)\,dA = \int_0^1 \int_0^2 (4 - y^2)\,dy\,dx = \int_0^1 \left[4y - \frac{y^3}{3}\right]_0^2 dx$
$\displaystyle= \int_0^1 \frac{16}{3}\,dx = \frac{16}{3}.$ ◀

32. Evaluate $\displaystyle\int_{-1}^1 \int_0^{\pi/2} x \sin\sqrt{y}\,dy\,dx.$

Solution. $\displaystyle\int_{-1}^1 \int_0^{\pi/2} x \sin\sqrt{y}\,dy\,dx = \int_{-1}^1 x\left[2\sin\sqrt{y} - 2\sqrt{y}\cos\sqrt{y}\right]_0^{\pi/2} dx$
$\displaystyle= \int_{-1}^1 x(2\sin\sqrt{\pi/2} - \sqrt{2\pi}\cos\sqrt{\pi/2}\,dx = (2\sin\sqrt{\pi/2} - \sqrt{2\pi}\cos\sqrt{\pi/2})\left[x^2\right]_{-1}^1 = 0.$

Alternatively, by Fubini's Theorem, $\displaystyle\int_{-1}^1 \int_0^{\pi/2} x \sin\sqrt{y}\,dy\,dx = \int_0^{\pi/2} \int_{-1}^1 x \sin\sqrt{y}\,dx\,dy$
$\displaystyle= \int_0^{\pi/2} \left[\frac{x^2 \sin\sqrt{y}}{2}\right]_{-1}^1 dy = \int_0^{\pi/2} 0\,dy = 0.$ ◀

34. Use Fubini's Theorem to evaluate

$$\int_0^1 \int_0^3 xe^{xy}\,dx\,dy.$$

Solution. By Fubinis's Theorem, $\displaystyle\int_0^1 \int_0^3 xe^{xy}\,dx\,dy = \int_0^3 \int_0^1 xe^{xy}\,dy\,dx = \int_0^3 \left[e^{xy}\right]_0^1 dx$
$\displaystyle= \int_0^3 (e^x - 1)\,dx = \left[e^x - x\right]_0^3 = e^3 - 4.$ ◀