MATH2010 Advanced Calculus I

Solution to Homework 5

§14.3

Q12

Solution.

$$f_x = \frac{-yx^{-2}}{1+y^2x^{-2}} = \frac{-y}{x^2+y^2}$$
$$f_y = \frac{x^{-1}}{1+y^2x^{-2}} = \frac{x}{x^2+y^2}$$

□

Q18

Solution.

$$f_x = 2\cos\big(3x-y^2\big)\big(-\sin\big(3x-y^2\big)\big)\cdot 3 = -6\sin\big(3x-y^2\big)\cos\big(3x-y^2\big) = -3\sin\big(6x-2y^2\big)$$
$$f_y = 2\cos\big(3x-y^2\big)\big(-\sin\big(3x-y^2\big)\big)\cdot(-2y) = 4y\sin\big(3x-y^2\big)\cos\big(3x-y^2\big) = 2y\sin\big(6x-2y^2\big)$$

□

Q27

Solution.

$$f_x = (1-x^2y^2z^2)^{-1/2}yz$$
$$f_y = (1-x^2y^2z^2)^{-1/2}xz$$
$$f_z = (1-x^2y^2z^2)^{-1/2}xy$$

□

Q37

Solution.

$$h_\rho = \sin\phi\cos\theta,\ h_\phi = \rho\cos\phi\cos\theta,\ h_\theta = -\rho\sin\phi\sin\theta$$

□

Q46

Solution.
As in Q12, $f = s$, then

$$s_{xx} = \frac{2xy}{(x^2+y^2)^2},\ s_{xy} = \frac{-(x^2+y^2)+y\cdot 2y}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$
$$s_{yx} = \frac{x^2+y^2-x\cdot 2x}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2},\ s_{yy} = \frac{-2xy}{(x^2+y^2)^2}$$

□

Q54

Solution.

$$w_x = \sin y + y\cos x + y,\ w_y = x\cos y + \sin x + x$$
$$w_{xy} = \cos y + \cos x + 1 = w_{yx}$$

□

Q60

Solution.

$$f_x(0,0) = \lim_{h\to 0}\frac{f(h,0)-f(0,0)}{h} = \lim_{h\to 0}\frac{\frac{\sin(h^3)}{h^2}}{h} = 1$$
$$f_y(0,0) = \lim_{h\to 0}\frac{f(0,h)-f(0,0)}{h} = \lim_{h\to 0}\frac{\frac{\sin(h^4)}{h^2}}{h} = \lim_{h\to 0}\frac{\sin(h^4)}{h^4}h = 0$$

□

Q65

Solution.

$$xy + z^3x - 2yz = 0$$

By implicit differentiation with respect to x,

$$y + 3z^2z_xx + z^3 - 2yz_x = 0$$

At the point $(1,1,1)$,

$$1 + 3\cdot 1^2z_x\cdot 1 + 1^3 - 2\cdot 1\cdot z_x = 0$$
$$z_x = -2$$

□

Q71

Solution.

$$f_x(x,y) = 0,\ \text{domain} = \mathbf{R}^2$$
$$f_y(x,y) = \begin{cases} 3y^2 & \text{if } y \geq 0 \\ -2y & \text{if } y < 0 \end{cases},\ \text{domain} = \mathbf{R}^2$$
$$f_{xy}(x,y) = 0,\ \text{domain} = \mathbf{R}^2$$
$$f_{yx}(x,y) = 0,\ \text{domain} = \mathbf{R}^2$$

□

Q80

Solution.

$$f_x = 3e^{3x+4y}\cos 5z$$
$$f_y = 4e^{3x+4y}\cos 5z$$
$$f_z = -5e^{3x+4y}\sin 5z$$
$$f_{xx} = 9e^{3x+4y}\cos 5z$$
$$f_{yy} = 16e^{3x+4y}\cos 5z$$
$$f_{zz} = -25e^{3x+4y}\cos 5z$$

Hence,

$$f_{xx} + f_{yy} + f_{zz} = 9e^{3x+4y}\cos 5z + 16e^{3x+4y}\cos 5z - 25e^{3x+4y}\cos 5z = 0.$$

Therefore, f satisfies a Laplace equation.

□