MATH2010 Advanced Calculus I

Solution to Homework 2

§12.5 **Q**33

Solution.

The line passes through $P(0,0,0)$ parallel to $\mathbf{v} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.
Let $S = (0,0,12)$.
$\overrightarrow{PS} = (0-0)\mathbf{i} + (0-0)\mathbf{j} + (12-0)\mathbf{k} = 12\mathbf{k}$

$$\overrightarrow{PS} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 4 & -2 & 2 \end{vmatrix} = 24\mathbf{i} + 48\mathbf{j}$$

$$\text{Distance} = \frac{|\overrightarrow{PS} \times \mathbf{v}|}{|\mathbf{v}|} = \frac{\sqrt{24^2 + 48^2}}{\sqrt{4^2 + 2^2 + 2^2}} = \frac{24\sqrt{5}}{2\sqrt{6}} = 2\sqrt{30}$$

□

§12.5 **Q**39

Solution.

$\mathbf{n} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ normal to the plane.
$P(13,0,0)$ is a point on the plane.
Let $S = (2,-3,4)$.
$\overrightarrow{PS} = (2-13)\mathbf{i} + (-3-0)\mathbf{j} + (4-0)\mathbf{k} = -11\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$
$|\mathbf{n}| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3$

$$\text{Distance} = \left|\overrightarrow{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|}\right| = \left|\frac{-11-6+8}{3}\right| = |-3| = 3$$

□

§12.5 **Q**57

Solution.

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -\mathbf{i} + \mathbf{j} \text{ parallel to the line of intersection.}$$

Consider $x = 1$. Then $y = 1$ and $z = -1$. Hence, $(1,1,-1)$ is a point on the line of intersection.
Therefore, the line is
$x = 1 - t, y = 1 + t, z = -1$.

□

§13.1 **Q**19

Solution.

$\mathbf{r}(t_0) = \mathbf{r}(0) = -\mathbf{j} + \mathbf{k}$
$\mathbf{v}(t) = (\cos t)\mathbf{i} + (2t + \sin t)\mathbf{j} + (e^t)\mathbf{k}$

$\mathbf{v}(t_0) = \mathbf{v}(0) = \mathbf{i} + \mathbf{k}$
Therefore, the tangent line is
$x = t, y = -1, z = 1 + t$.

□

§13.3 **Q**2

Solution.

$\mathbf{v} = (12\cos 2t)\mathbf{i} + (-12\sin 2t)\mathbf{j} + (5)\mathbf{k}$
$|\mathbf{v}| = \sqrt{(12\cos 2t)^2 + (-12\sin 2t)^2 + 5^2} = \sqrt{12^2 + 5^2} = 13$
Hence, the unit tangent vector is
$\mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = \left(\frac{12}{13}\cos 2t\right)\mathbf{i} - \left(\frac{12}{13}\sin 2t\right)\mathbf{j} + \frac{5}{13}\mathbf{k}$
Length $= \int_0^{\pi} |\mathbf{v}| dt = \int_0^{\pi} 13 dt = 13t\Big|_0^{\pi} = 13\pi$

□

§13.3 **Q**18

Solution.

a. $\mathbf{v} = (-4\sin 4t)\mathbf{i} + (4\cos 4t)\mathbf{j} + 4\mathbf{k}$
   $|\mathbf{v}| = \sqrt{4^2 + 4^2} = 4\sqrt{2}$
   Length $= \int_0^{\pi/2} |\mathbf{v}| dt = \int_0^{\pi/2} 4\sqrt{2} dt = 4\sqrt{2}t\Big|_0^{\pi/2} = 2\sqrt{2}\pi$

b. $\mathbf{v} = \left(-\frac{1}{2}\sin(t/2)\right)\mathbf{i} + \left(\frac{1}{2}\cos(t/2)\right)\mathbf{j} + (1/2)\mathbf{k}$
   $|\mathbf{v}| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{\sqrt{2}}{2}$
   Length $= \int_0^{4\pi} |\mathbf{v}| dt = \int_0^{4\pi} \frac{\sqrt{2}}{2} dt = \frac{\sqrt{2}}{2}t\Big|_0^{4\pi} = 2\sqrt{2}\pi$

c. $\mathbf{v} = (-\sin t)\mathbf{i} - (\cos t)\mathbf{j} - \mathbf{k}$
   $|\mathbf{v}| = \sqrt{1^2 + 1^2} = \sqrt{2}$
   Length $= \int_{-2\pi}^{0} |\mathbf{v}| dt = \int_{-2\pi}^{0} \sqrt{2} dt = \sqrt{2}t\Big|_{-2\pi}^{0} = 0 - (-2\sqrt{2}\pi) = 2\sqrt{2}\pi$

The length computed using all these parametrization are the same as Example 1.

□

§11.3 **Q**27

Solution.

The equivalent Cartesian equation is $x = 2$.
The graph is a vertical line through $x = 2$ on the x-axis.

□

§11.3 **Q**59

Solution.

$\frac{x^2}{9} + \frac{y^2}{4} = 1$

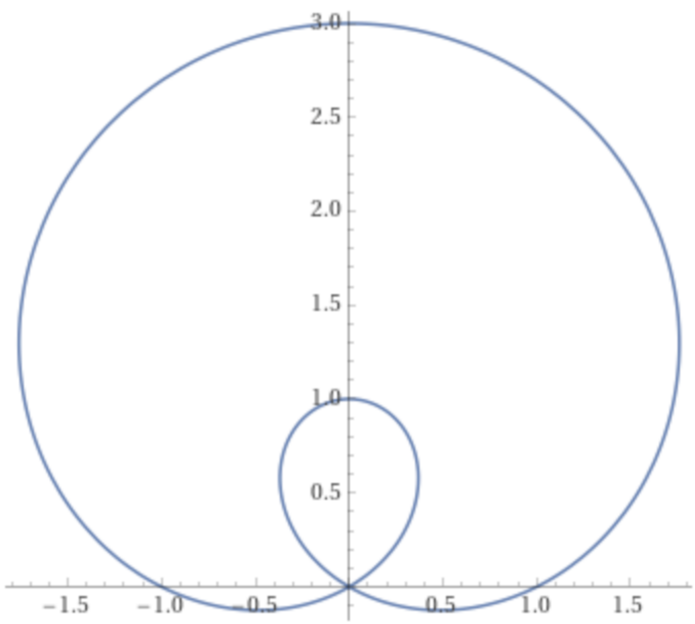
$\dfrac{r^2\cos^2\theta}{9} + \dfrac{r^2\sin^2\theta}{4} = 1$
$4r^2\cos^2\theta + 9r^2\sin^2\theta = 36$

□

§11.4 **Q6**

Solution.

1. Symmetry about the x-axis:
   If (r,θ) on the graph, $r = 1 + 2\sin\theta$.
   $1 + 2\sin(-\theta) = 1 - 2\sin\theta \neq r$, hence $(r,-\theta)$ is not on the graph.
   $1 + 2\sin(\pi - \theta) = 1 + 2\sin\theta = r \neq -r$, hence $(-r, \pi - \theta)$ is not on the graph.
   Therefore, no symmetry about the x-axis.
2. Symmetry about the y-axis:
   If (r,θ) on the graph, $r = 1 + 2\sin\theta$.
   $1 + 2\sin(\pi - \theta) = 1 + 2\sin\theta = r$, hence $(r, \pi - \theta)$ is on the graph.
   Therefore, the curve is symmetric about the y-axis.
3. Hence, no symmetry about the origin.



□

§11.4 **Q19**

Solution.

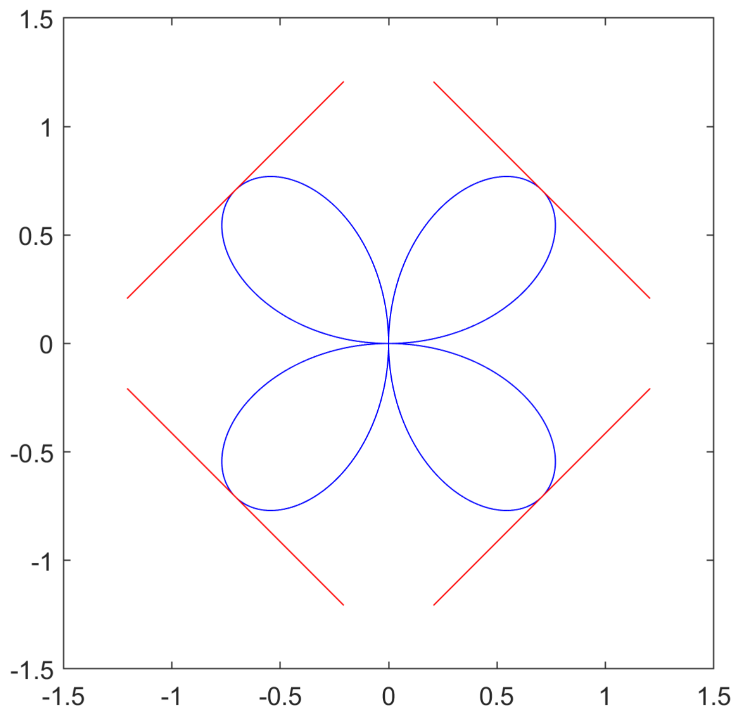
$r = \sin 2\theta$
$r' = 2\cos 2\theta$
$\left.\dfrac{dy}{dx}\right|_{(r,\theta)} = \dfrac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$

Hence,
at $\theta = \pi/4$:
$r = \sin(\pi/2) = 1,\ r' = 2\cos(\pi/2) = 0$
Slope $= \left.\dfrac{dy}{dx}\right|_{(1,\pi/4)} = \dfrac{\cos(\pi/4)}{-\sin(\pi/4)} = -1$

at $\theta = -\pi/4$:
$r = \sin(-\pi/2) = -1,\ r' = 2\cos(-\pi/2) = 0$
$$\text{Slope} = \left.\frac{dy}{dx}\right|_{(-1,-\pi/4)} = \frac{-\cos(-\pi/4)}{\sin(-\pi/4)} = 1$$

at $\theta = 3\pi/4$:
$r = \sin(3\pi/2) = -1,\ r' = 2\cos(3\pi/2) = 0$
$$\text{Slope} = \left.\frac{dy}{dx}\right|_{(-1,3\pi/4)} = \frac{-\cos(3\pi/4)}{\sin(3\pi/4)} = 1$$

at $\theta = -3\pi/4$:
$r = \sin(-3\pi/2) = 1,\ r' = 2\cos(-3\pi/2) = 0$
$$\text{Slope} = \left.\frac{dy}{dx}\right|_{(1,-3\pi/4)} = \frac{\cos(-3\pi/4)}{-\sin(-3\pi/4)} = -1$$



□