MATH2010 Advanced Calculus I

Solution to Homework 4

14.2

Let $\mathbb{R}^2 = \{(x, y) |\, x, y \text{ are real number.}\}$.

31. a.$\mathbb{R}^2$.   b. $\mathbb{R}^2 \backslash (0, 0)$.

33. a.$\mathbb{R}^2 \backslash \left(\{x = 0\} \cup \{y = 0\}\right)$.   b. $\mathbb{R}^2$.

Let $\mathbb{R}^3 = \{(x, y, z) |\, x, y, z \text{ are real number.}\}$.

36. a.$\mathbb{R}^3 \backslash \{xyz \leq 0\}$.   b. $\mathbb{R}^3$.

38. a.$\mathbb{R}^3 \backslash \left(\{y = z = 0\}\right)$.   b. $\mathbb{R}^3 \backslash \left(\{x = z = 0\} \cup \{y = z = 0\}\right)$.

60. By

$$0 \leq \left| xy \frac{x^2 - y^2}{x^2 + y^2} \right| \leq \sqrt{x^2 + y^2} \cdot \sqrt{x^2 + y^2} \cdot \frac{x^2 + y^2}{x^2 + y^2} = x^2 + y^2 \to 0, \quad \text{as } (x, y) \to (0, 0)$$

and the Sandwich Theorem, we can take $f(0, 0) = 0$.

61. By changing to polar coordinates, we have

$$f(r\cos\theta, r\sin\theta) = \frac{r^3 \cos\theta(\cos^2\theta - \sin^2\theta)}{r^2} \to 0, \qquad \text{as } r \to 0.$$

63. By changing to polar coordinates, we have

$$f(r\cos\theta, r\sin\theta) = \frac{r^2\sin^2\theta}{r^2} = \sin^2\theta,$$

hence the limit does not exist.

67. By changing to polar coordinates, we have

$$f(r\cos\theta, r\sin\theta) = \ln\left(3 - \frac{r^4\cos^2\theta\sin^2\theta}{r^2}\right) = \ln 3, \qquad \text{as } r \to 0.$$

hence we can take $f(0,0) = \ln 3$.

70. When $\sqrt{x^2+y^2} < \epsilon$, we have

$$|f(x,y) - f(0,0)| \leq |y| \leq \sqrt{x^2+y^2} < \epsilon.$$

76. When $\sqrt{x^2+y^2+z^2} < \epsilon = 0.008 < 1$, we have

$$|f(x,y,z) - f(0,0,0)| \leq |x| \leq \sqrt{x^2+y^2+z^2} < \epsilon.$$