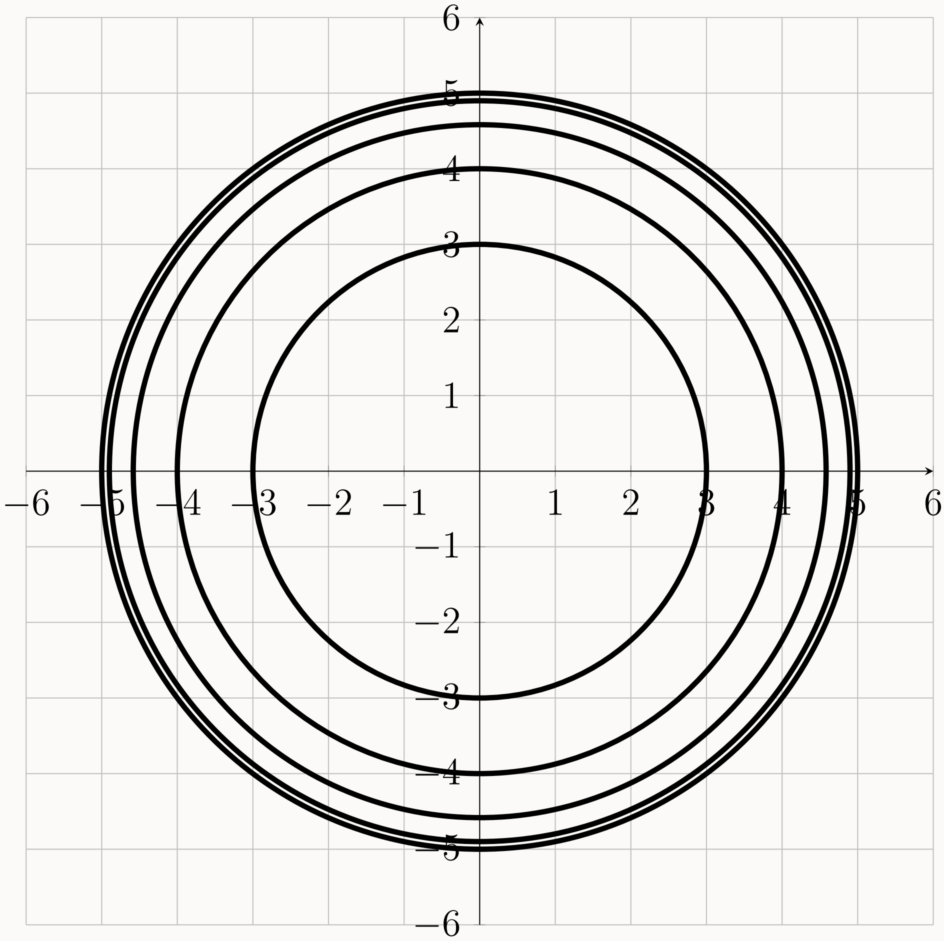
MATH2010 Advanced Calculus I

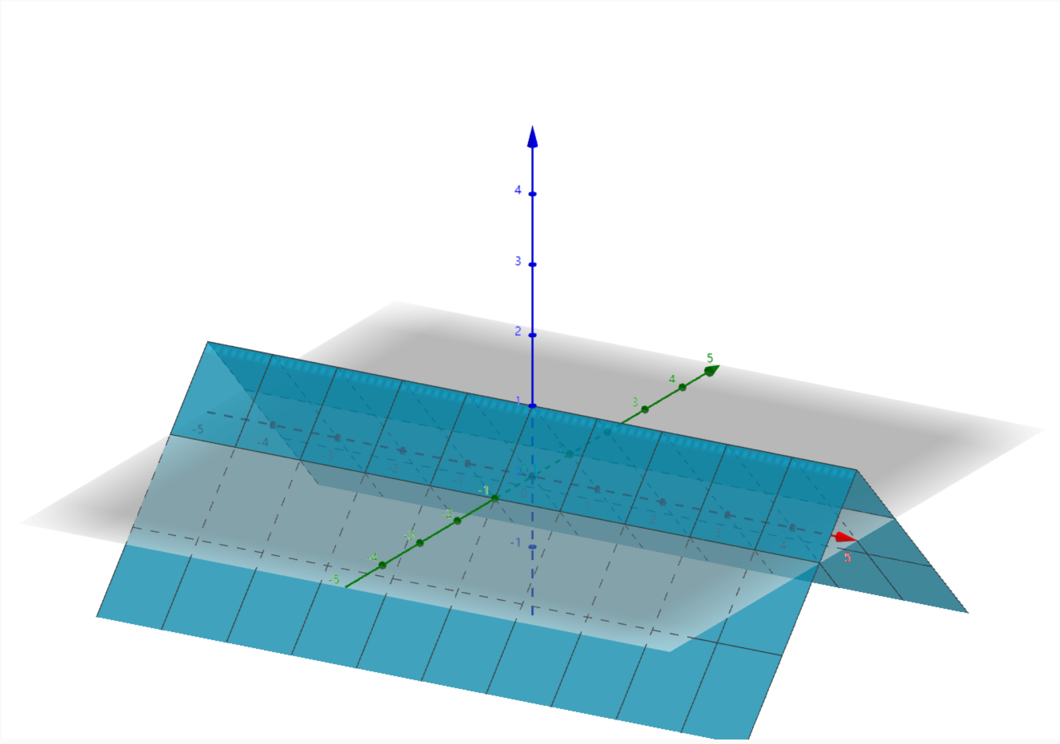
Solution to Homework 3

14.1

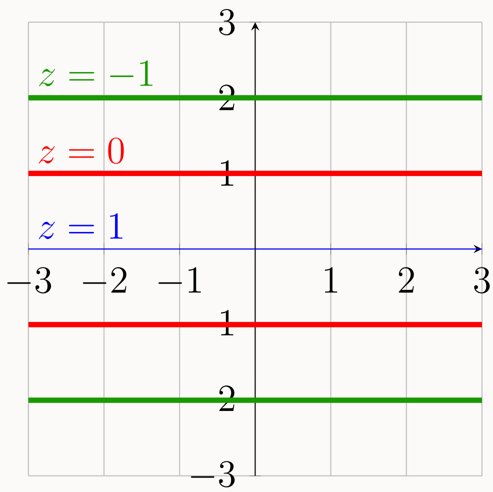
16. The level curve:



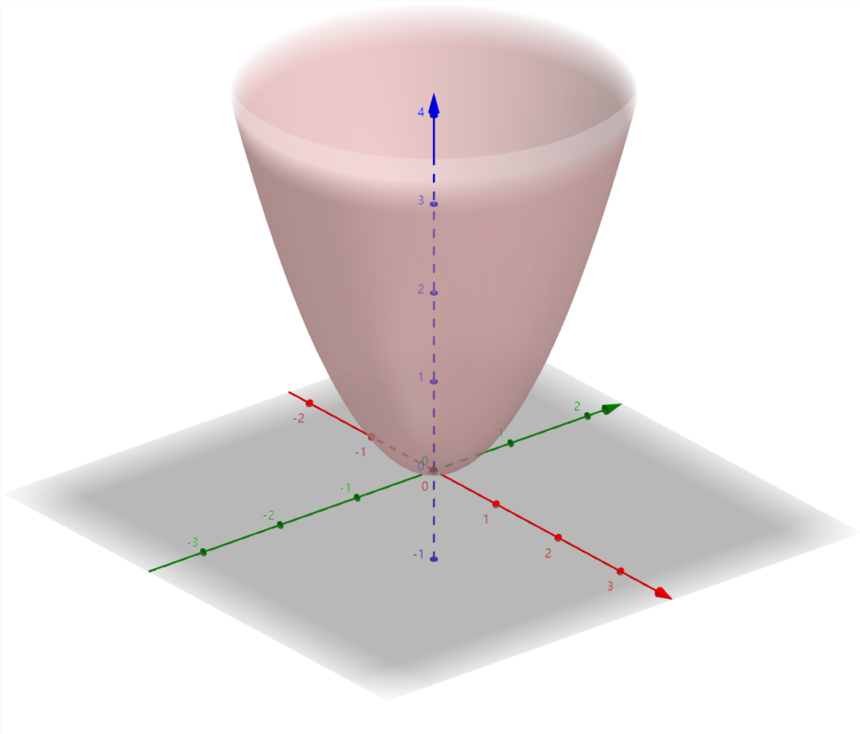
45. The surface:



The level curve:



59. The level surface $f(x, y, z) = z - x^2 - y^2 = 0$:



61. By

$$f(3, -1, 1) = \sqrt{3 - (-1)} - \ln 1 = 2,$$

the level surface of f through $(3, -1, 1)$ is

$$\sqrt{x - y} - \ln z = 2.$$

14.2

10. $\lim_{(x,y)\to(1/27,\pi^3)} \cos \sqrt[3]{xy} = \cos(\pi/3) = 1/2$

20.

$$\begin{aligned}
&\lim_{(x,y)\to(4,3),x\neq y+1} \frac{\sqrt{x}-\sqrt{y+1}}{x-y-1} \\
&= \lim_{(x,y)\to(4,3),x\neq y+1} \frac{\sqrt{x}-\sqrt{y+1}}{(\sqrt{x}-\sqrt{y+1})(\sqrt{x}+\sqrt{y+1})} \\
&= \lim_{(x,y)\to(4,3),x\neq y+1} \frac{1}{\sqrt{x}+\sqrt{y+1}} \\
&= \frac{1}{\sqrt{4}+\sqrt{3+1}} = \frac{1}{4}
\end{aligned}$$

29. $\lim\limits_{P\to(\pi,0,3)} ze^{-2y}\cos 2x = 3e^0\cos 2\pi = 3$

42. $\lim\limits_{(x,y)\to(0,0),y=x^2} \dfrac{x^4}{x^4+y^2} = \lim\limits_{x\to 0} \dfrac{x^4}{2x^4} = \dfrac{1}{2} \neq \lim\limits_{(x,0)\to(0,0)} \dfrac{x^4}{x^4+y^2} = \lim\limits_{x\to 0} \dfrac{x^4}{x^4} = 1$

52. a. $\lim\limits_{(x,y)\to(3,-2)} f(x,y) = \lim\limits_{(x,y)\to(3,-2)} x^2 = 9$

b. $\lim\limits_{(x,y)\to(-2,1)} f(x,y) = \lim\limits_{(x,y)\to(-2,1)} x^3 = -8$

c. $\lim\limits_{(x,y)\to(0,0),x\geq 0} f(x,y) = \lim\limits_{(x,y)\to(0,0),x\geq 0} x^2 = 0,$

$\lim\limits_{(x,y)\to(0,0),x<0} f(x,y) = \lim\limits_{(x,y)\to(0,0),x<0} x^3 = 0,$

hence $\lim\limits_{(x,y)\to(0,0)} f(x,y) = 0.$

57. We can get the limit. $|\sin(1/x)| \leq 1 \implies -y < y\sin\dfrac{1}{x} < y$, then by

$$\lim_{(x,y)\to(0,0)} -y = 0 = \lim_{(x,y)\to(0,0)} y$$

and The Sandwich Theorem, we have $\lim\limits_{(x,y)\to(0,0)} y\sin\dfrac{1}{x} = 0.$