3.4.2 Answers to Exercise.

1. (a) $x = 7$, $y = 4$, $z = -4$.
   (b) $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ are linearly dependent.
   (c) $u = 1$, $v = 0$, $w = 4$.
2. (a) ——
   (b) ——
   (c) One possible choice is to take $C = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}$, $H = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.
   (d) One possible choice is to take $A = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$, $G = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.
3. (a) True.
   (b) True.
   (c) True.
   (d) True.
   (e) True.
4. ——
5. ——
6. (a) ——
   (b) i. $A = GJ_3^{3,4}H$, in which $G = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 3 \\ 2 & 6 & 5 \end{bmatrix}$, $H = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

   ii. $A = GJ_3^{3,4}H$, in which $G = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -2 & 7 \\ -1 & 3 & 3 \end{bmatrix}$, $H = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

   iii. $A = GJ_2^{3,4}H$, in which $G = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -2 & 0 \\ 2 & 7 & 1 \end{bmatrix}$, $H = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

   iv. $A = GJ_2^{3,5}H$, in which $G = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$, $H = \begin{bmatrix} 1 & 0 & 2 & -3 & -1 \\ 0 & 1 & -1 & 2 & 4 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$.

   v. $A = GJ_3^{3,6}H$, in which $G = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 2 \\ -2 & -1 & 3 \end{bmatrix}$, $H = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 10 \\ 0 & 1 & 1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$.

   vi. $A = GJ_3^{4,5}H$, in which $G = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 3 & 2 & -1 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$, $H = \begin{bmatrix} 1 & 0 & -1 & 0 & 3 \\ 0 & 1 & 4 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$.