2.3.2 Answers to Exercise.

1. (a) *Comment.*

   Make use of the 'dictionary' betwen application of row operations and left-multiplication by row-operation matrices.

   (b) *Comment.*

   Make use of the 'dictionary' between application of a sequence of row operations and left multiplication by products of row operation matrices.

   Also make use of the result on the existence of row-echelon forms and a reduced row-echelon form which are row-equivalent to an arbitrarily given matrix.

2. (a) $C' = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$.

   $H = \begin{bmatrix} 3 & -2 & 0 \\ -1 & -1 & 1 \\ 0 & 2 & -1 \end{bmatrix}$.

   (b) $C' = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

   $H = \begin{bmatrix} -9 & 3 & -1 \\ -16/3 & 5/3 & -1/3 \\ 7/3 & -2/3 & 1/3 \end{bmatrix}$.

   (c) $C' = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

   $H = \begin{bmatrix} -2 & -1 & 0 \\ 1 & 0 & 0 \\ -3 & 2 & 1 \end{bmatrix}$.

   (d) $C' = \begin{bmatrix} 1 & 0 & 2 & -3 & -1 \\ 0 & 1 & -1 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

   $H = \begin{bmatrix} -1 & 2 & 0 \\ 1 & -1 & 0 \\ 2 & -5 & 1 \end{bmatrix}$.

   (e) $C' = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 10 \\ 0 & 1 & 1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 1 & 1 & 5 \end{bmatrix}$.

   $H = \begin{bmatrix} -8 & 5 & 2 \\ 7 & -4 & -2 \\ -3 & 2 & 1 \end{bmatrix}$.

   (f) $C' = \begin{bmatrix} 1 & 0 & -1 & 0 & 3 \\ 0 & 1 & 4 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

   $H = \begin{bmatrix} -3 & 5 & 0 & -1 \\ 1 & -1 & 0 & 0 \\ 2 & -3 & 0 & 1 \\ 9 & -16 & 1 & 4 \end{bmatrix}$.

3. (a) $B^\sharp = [\ A^\sharp \mid A^\sharp\ ]$.

   $B' = [\ A' \mid A'\ ]$.

   $B^\sharp = JB$.

   $B' = KB^\sharp$.

   (b) $C^\sharp = \left[\begin{array}{c} A^\sharp \\ \hline \mathcal{O}_{p\times q} \end{array}\right]$.

   $C' = \left[\begin{array}{c} A' \\ \hline \mathcal{O}_{p\times q} \end{array}\right]$.

   $C^\sharp = \left[\begin{array}{c|c} J & \mathcal{O}_{p\times p} \\ \hline \mathcal{O}_{p\times p} & I_p \end{array}\right] \left[\begin{array}{c|c} I_p & \mathcal{O}_{p\times p} \\ \hline -I_p & I_p \end{array}\right] C$.

   $C' = \left[\begin{array}{c|c} K & \mathcal{O}_{p\times p} \\ \hline \mathcal{O}_{p\times p} & I_p \end{array}\right] C^\sharp$.

(c) $D^\sharp = \left[\begin{array}{c|c} A^\sharp & \mathcal{O}_{p\times q} \\ \hline \mathcal{O}_{p\times q} & A^\sharp \end{array}\right].$

$D' = \left[\begin{array}{c|c} A' & \mathcal{O}_{p\times q} \\ \hline \mathcal{O}_{p\times q} & A' \end{array}\right].$

$D^\sharp = \left[\begin{array}{c|c} J & \mathcal{O}_{p\times p} \\ \hline \mathcal{O}_{p\times p} & J \end{array}\right] D.$

$D' = \left[\begin{array}{c|c} K & \mathcal{O}_{p\times p} \\ \hline \mathcal{O}_{p\times p} & K \end{array}\right] D^\sharp.$

(d) $E^\sharp = \left[\begin{array}{c|c} A^\sharp & A^\sharp \\ \hline \mathcal{O}_{p\times q} & A^\sharp \end{array}\right].$

$E' = \left[\begin{array}{c|c} A' & \mathcal{O}_{p\times q} \\ \hline \mathcal{O}_{p\times q} & A' \end{array}\right].$

$E^\sharp = \left[\begin{array}{c|c} J & \mathcal{O}_{p\times p} \\ \hline \mathcal{O}_{p\times p} & J \end{array}\right] E.$

$E' = \left[\begin{array}{c|c} I_p & -I_p \\ \hline \mathcal{O}_{p\times p} & I_p \end{array}\right] \left[\begin{array}{c|c} K & \mathcal{O}_{p\times p} \\ \hline \mathcal{O}_{p\times p} & K \end{array}\right] E^\sharp.$