1.6.1 Answers to Exercise.

1. (a) *Comment.*
   The equality $1 \cdot \mathbf{0}_p = \mathbf{0}_p$ is involved.

   (b) *Comment.*
   The equality $1 \cdot \mathbf{0}_p + 0 \cdot \mathbf{u}_1 + 0 \cdot \mathbf{u}_2 + \cdots + 0 \cdot \mathbf{u}_q = \mathbf{0}_p$ is involved.

   (c) *Comment.*
   The equality $\beta \cdot \alpha\mathbf{u} - \alpha \cdot \beta\mathbf{u} = \mathbf{0}_p$ is involved.

   (d) *Comment.*
   The equality $(-1)\mathbf{u} + 1 \cdot \mathbf{v} + 1 \cdot (\mathbf{u} - \mathbf{v}) = \mathbf{0}_p$ is involved.

   (e) *Comment.*
   The equality $(-\beta\gamma)\mathbf{u} + (-\alpha\delta)\beta\mathbf{v}\alpha\beta(\gamma\mathbf{u} + \delta\mathbf{v}) = \mathbf{0}_p$ is involved.

   (f) ——

2. ——
3. ——
4. ——
5. (a) ——

   (b) i. A possible counter-example is provided by $\mathbf{t} = \begin{bmatrix} -1/2 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

   ii. ——

6. ——
7. (a) i. $\beta_1 = 2\alpha_1 + \alpha_2$, $\beta_2 = \alpha_1 - \alpha_2 + \alpha_3$, $\beta_3 = -3\alpha_1 + \alpha_3$.

   ii. ——

   (b) i. $\mathbf{u}_2 = \mathbf{u}_1 - \mathbf{v}_2 = \frac{1}{6}\mathbf{v}_1 - \frac{1}{3}\mathbf{v}_2 + \frac{1}{2}\mathbf{v}_3$.

   ii. ——